A PROCEDURE FOR PREDICTING FUEL SPECIFIC IMPULSE OF SUPERSONIC COMBUSTION RAMJET ENGINES †

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INTRODUCTION section of this report is written by Professor Jack L. Kerrebrock.

ABSTRACT

This report presents a procedure in order to portray the effect of design choices on SCRAMJET dynamic performance in the context of real gas effects. Procedure is based on one-dimensional fluid flow analysis. The less quantifiable aspects of boundary layer transition, mixing, and non-uniform nozzle flow are included in approximate parametric fashion by allowing for parallel flow paths through the engine and nozzle, with possibility of thermal energy exchange between various flow streams. The effect of finite recombination rates in the nozzle is included by allowing for freezing of the chemical composition at any specified static pressure in the nozzle.

Exploratory results from the digital implementation of this procedure in FORTRAN language are also included.

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LIST OF SYMBOLS

Superscripts:

o = quantity at standard pressure of 1 bar

$*$ = free stream quantities

Subscripts for SCRAMJET stations (i):

0 = diffuser inlet

H = fuel source exit

$1X$ = fuel energy augmenter exit ($1X = H$ in the absence of augmenter)

1 = fuel at heat exchanger exit ($1 = 1X$ in the absence of heat exchanger)

$2X$ = diffuser exit

2 = air at heat exchanger exit ($2 = 2X$ in the absence of heat exchanger)

3 = fuel at mixer inlet or unmixed fuel nozzle inlet or bypass fuel nozzle inlet ($3 = 1$)

4 = mixer exit or reactor inlet

$5X$ = reactor exit

5 = reacting stream nozzle inlet ($5 = 5X$ in the absence of heat exchanger)

$5'$ = unmixed air stream nozzle inlet ($5' = 2$)

$5''$ = unmixed fuel stream nozzle inlet ($5'' = 3 = 1$)

$5'''$ = air bypass stream nozzle inlet ($5''' = 2X$)

5^{IV} = fuel bypass stream nozzle inlet ($5^{IV} = 3 = 1$)

6 = reacting stream nozzle exit

$6'$ = unmixed air stream nozzle exit

$6''$ = unmixed fuel stream nozzle exit

$6'''$ = air bypass stream nozzle exit

6^{IV} = fuel bypass stream nozzle exit

Symbols:

A_D = vehicle drag characteristic area

A_i = cross-sectional area at station i

A_0^* = vehicle free-stream capture area

a_i = speed of sound at station i

a_0^* = speed of sound in free stream

C_D = reactor drag coefficient

C_e = p_e / p_0^* = nozzle expansion coefficient

C_F = thrust coefficient

C_f = vehicle skin friction coefficient

$(C_p)_i$ = constant pressure specific heat at station i

$[\bar{C}_p]_j$ = constant pressure molar specific heat of j th chemical species

C_U = nozzle velocity coefficient (See equation (40))

$D(t)$ = vehicle drag force

f_e = $\dot{m}_3' / \dot{m}_2'$ = effective fuel air mass ratio

f_m = $\dot{m}_3 / m_2$ = mixer fuel-air mass ratio

f_o = $\dot{m}_H / \dot{m}_o$ = overall fuel-air mass ratio

f_x = heat exchanger fuel-air mass ratio (See equations (47) and (49))

g = acceleration of gravity

h_i = complete enthalpy at station i (See equation (16))

$[\Delta\bar{h}(T)]_j$ = molar enthalpy change of j th chemical species from reference temperature, $T_r = 298.15\ K$ to temperature, T.

$[\bar{h}_f^o(T_r)]_j$ = molar enthalpy of formation of j th chemical species at reference temperature, $T_r = 298.15\ K$ and at standard pressure, p^o = 1 bar.

H_i = $h_i + \frac{1}{2} U_i^2$ = complete energy per unit mass at station i

$(I_{sp})_{air}$ = $T / \dot{m}_0^*$ = air specific impulse

$(I_{sp})_{fuel}$ = $T / \dot{m}_H$ = fuel specific impulse

$\mathcal{M}_i$ = molecular weight at station i

$\mathcal{M}_j$ = molecular weight of j^{th} chemical species

M_0^* = flight Mach number

M_i = Mach number at station i

$m(t)$ = mass of vehicle at time, t

$\dot{m}_i$ = mass flow rate at station i

$\dot{m}_0^*$ = $\rho_0^* U_0^* A_0^*$ = rate of air mass capture

$\dot{m}_2'$ = air mass flow rate entering perfect mixer

$\dot{m}_3'$ = fuel mass flow rate entering perfect mixer

$(n_T)_i$ = number of moles at station i

p_0^* = free-stream static pressure

p_i = static pressure at station i

p_e = nozzle exit static pressure

p_{ti} = stagnation pressure at station i

$\dot{Q}_H$ = rate of heat energy input at fuel energy augmenter

$\dot{Q}_X$ = rate heat energy input at heat exchanger

q_i = $\frac{1}{2} \rho_i U_i^2$ = dynamic pressure at station i

q_0^* = free-stream dynamic pressure

R_0^* = free-stream gas constant

R_i = $\mathcal{R} / \mathcal{M}_i$ = gas constant at station i

$\mathcal{R}$ = universal gas constant

Re = Reynold's number

s_i = entropy at station i (See equation (19))

$[\bar{s}^\circ(T)]_j$ = molar entropy of j^{th} chemical species at static temperature, T and at standard pressure, p° = 1 bar.

T = thrust (See equation (13))

T_0^* = free-stream static temperature

T_i = static temperature at station i

T_r = $298.15\ K$ reference temperature

T_{ti} = stagnation temperature at station i

t = time

U_0^* = vehicle flight speed

U_i = speed at station i

y_j = mole fraction of j^{th} species

z_0^* = flight altitude

$z(t)$ = flight altitude at time, t

α = 0.2095 = mole fraction of O_2 in reference air

β_A = $\frac{1}{\eta_A} - 1$ = bypass ratio of unmixed air (an alternative measure for air mixing efficiency, η_A)

β_H = $\frac{1}{\eta_H} - 1$ = bypass ratio of unmixed fuel (an alternative measure for fuel mixing efficiency, η_H)

β_Q = air bypass ratio

β_R = fuel bypass ratio

γ_0^* = free-stream specific heat ratio

γ_i = $(c_p)_i / (c_v)_i$ = specific heat ratio at station i

ε = p_5 / p_6 = nozzle expansion ratio

η_A = $\dot{m}_2' / \dot{m}_2$ = air mixing efficiency

η_D = diffuser kinetic energy efficiency (See equation (27))

η_H = $\dot{m}_3' / \dot{m}_3$ = fuel mixing efficiency

η_X = a measure of effectiveness of heat exchanger (See equations (48) and 50))

μ_0^* = free-stream absolute viscosity

ρ_0^* = free-stream density

ρ_i = density at station i

$\phi \doteq 34.29 f$ = equivalence ratio (an alternative measure of fuel-air mass ratio, f)

ϕ_e = effective equivalence ratio

ϕ_m = mixer equivalence ratio

ϕ_o = overall equivalence ratio

INTRODUCTION

With current interest in the supersonic combustion ramjet engine, stimulated by its central role as the high speed propulsion system in the National Aerospace Plane concept, there is a need for accurate estimation of the performance of such SCRAMJETS. Several factors make such estimates difficult at present. The most fundamental stems from the unique physical characteristics of the supersonic (or hypersonic) flow through the engine. The point of transition of the boundary layer on the inlet ramp is quite uncertain, making the structure of the flow which must be ingested by the engine difficult to predict. Mixing of the hydrogen fuel with the supersonic flow in the combustor section of the engine is a complex process, only partially predictable. The flow of the resultant non-uniform products through the exhaust nozzle is complicated by kinetic effects, as well as by possible thermal or composition stratification. These aspects of the SCRAMJET are all the subject of active research; many of the critical questions will be answered only by flight research. The estimation of SCRAMJET performance is also complicated by the real gas behavior of air and of combustion products at the very high temperatures which arise in the flow. The very concept of the SCRAMJET stems from the need to reduce the temperature in the combustion process so as to reduce the level of dissociation of the combustion products, which has the effect of blocking the conversion of chemical energy to thermal energy, unless there is effective recombination in the nozzle. These effects are quantifiable, but the numerical calculations required are somewhat complex, sufficiently so that there are to the author's knowledge no readily available systematic parametric studies of the SCRAMJET engine which show the effect of real gas behavior over a wide range of flight Mach number and broad set of design choices.

The purpose of this work is to provide such a parametric study and capability, showing the effects of real gas behavior. The less quantifiable aspects of boundary layer transition, mixing, and non-uniform nozzle flows will be included in approximate parametric fashion by allowing for parallel flow paths through the engine and nozzle, with the possibility of thermal energy exchange between the various flow streams. The effect of finite recombination rates in the nozzle is included by allowing for freezing of the chemical composition at any specified static pressure in the nozzle.

Since the principal objective is to portray the effect of design choices on SCRAMJET performance in the context of real gas effect, the results will be presented as a set of plots of fuel specific impulse versus flight Mach number, each for a different set of design choices. However, the computer program provides the gas properties throughout the flow path, and these results will be presented when deemed to be especially pertinent.

Finally, examples of dynamic behavior of a hypothetical vehicle with different design choices will be presented.

AN OVERVIEW OF SCRAMJET PROPULSION SYSTEM

A schematic diagram of a SCRAMJET propulsion system is presented in Figure 1. Primarily the propulsion system is visualized as to consist of an air bypass stream, a fuel bypass stream, and a reacting fuel-air stream. In order to model fuel-air mixing imperfections, the reacting fuel-air stream is subdivided into three streams: an unmixed air stream, an unmixed fuel stream, and a perfectly mixed fuel-air stream. In all streams the flow is assumed to be "one dimensional".

In air bypass and unmixed air streams the composition of gas consists of following species: O , N , NO , NO_2 , N_2O , O_2 , N_2 , and Ar . In fuel bypass and unmixed fuel streams the composition of gas consists of following species: H and H_2 . In reacting fuel-air stream the composition of gas consists of following species: O , H , OH , H_2O , N , NO , HO_2 , NO_2 , N_2O , O_2 , H_2 , N_2 , and Ar . Thus the present propulsion system analysis tries to capture essential characteristics of real gas glow.

Basic components of SCRAMJET propulsion system are a fuel energy augmenter, a diffuser, a combustor, and a set of nozzles. The role of fuel energy augmenter is to control the thermodynamic state of fuel which is introduced into the combustor. For the ease of system modeling the combustor is visualized to consist of a mechanical mixer followed by a reactor. It is to be noted that not all nozzles depicted in Figure 1 exist physically. At each propulsion system component inlet, the state of gas is considered to be in chemical equilibrium corresponding to its thermodynamic state. The flow through nozzles may correspond to either equilibrium or frozen composition flow. In addition, it is possible to freeze gas composition at a specified location within the nozzle. This location is identified by specifying the static pressure of the gas.

Various propulsion system analysis options are always reduced to equivalently specifying flight altitude, z_0^* and flight Mach number, M_0^* which are principal independent variables for a SCRAMJET in steady flight.

Temperature, T_0^* , pressure, p_0^* , and density, ρ_0^* of the air at specified altitude, z_0^* are determined via table look up and interpolation procedures from the U.S. Standard Atmosphere Tables, 1976 (Reference [1]). The flight speed, U_0^* of the vehicle then is

$$U_0^* = M_0^* a_0^* \tag{1}$$

where a_0^* is the speed of sound at the altitude, z_0^* namely

$$a_0^* = \sqrt{\gamma_0^* R_0^* T_0^*} \tag{2}$$

It is anticipated that temperature, T_0 , pressure p_0 , and speed, U_0 of air entering the diffuser are not the same as the temperature, T_0^* , pressure, p_0^* of the ambient air at flight altitude, z_0^* , and the flight speed, U_0^* . It is probable that ambient air may interact thermally with hot structure of the SCRAMJET and change its thermodynamic state. In this analysis T_0 may be specified to be different than T_0^* for some flight conditions, on the otherhand it is always assumed that $p_0 = p_0^*$ and $U_0 = U_0^*$.

The performance of diffuser is characterized by specifying its kinetic energy efficiency, η_D and its exit static pressure, p_2 indirectly since it is assumed that combustor inlet static pressure and diffuser exit static pressure are always the same. As implied in Figure 1, chemical and thermodynamic states of air in the air bypass stream are identical that of air at diffuser exit.

Liquid fuel source is characterized by specifying fuel exit static pressure and speed.

Primary role of fuel energy augmenter is to set fuel thermodynamic state appropriately in order to accomplish efficient mixing with air. Fuel energy augmenter always increases fuel energy and this energy is supplied by the combustor as implied in Figure 1. It is assumed that fuel at augmenter exit is in gaseous phase and its static temperature, static pressure and speed are specified.

Hypothetical mixer is lossless and is assumed to mix fuel and air mechanically, that is without chemical reaction. Mixture static temperature and speed are determined with the knowledge of reactor inlet static pressure and by using appropriate momentum and energy equations.

Reactor is characterized by specifying either pressure ratio or cross-sectional area ratio across it and a reactor drag coefficient, C_D to take into account reactor frictional losses. Thermodynamic state of combustion products are determined by using appropriate momentum and complete energy equations.

For the sake of simplicity in the performance analysis of SCRAMJET, Figure 1 considers a number of nozzles. Clearly, not all of these nozzles exist physically. Each nozzle conserves complete energy of expanding combustion products thus recombination of chemical species are considered whenever flow conditions dictate. Nozzle losses due to viscous effects are accounted by specifying a nozzle velocity coefficient, C_V . Nozzle exhaust pressure, p_e (where e

stands for 6, $6'$,..., 6^{IV} in Figure 1) is controlled by nozzle expansion coefficient, $C_e = p_e/p_0^*$. For an ideally expanded nozzle, Ce = 1.

It is anticipated that fuel shall be used to cool various SCRAMJET components before it is injected into the combustor. For this reason present analysis introduces a heat exchanger either at diffuser exit (See Figure 2) or at the reactor exit (See Figure 3). It is assumed that there are no pressure or viscous friction losses in heat exchangers and heat exchanging streams are isolated from each other physically.

As mentioned previously, principal independent variables of SCRAMJET performance analysis are flight Mach number, M_0^* and flight altitude, z_0^*. Flight speed, U_0^* may be used as independent variable instead of flight Mach number, M_0^*. Similarly, dynamic pressure, $q_0^* = \frac{1}{2}\gamma_0^* M_0^{*2} p_0^*$ or reactor inlet Mach number, M_4 may be used as an independent variable instead of flight altitude, z_0^*.

DEFINITIONS AND BASIC RELATIONSHIPS

Various Fuel-Air ratios: By definition overall fuel-air ratio, f_0 and mixer fuel-air ratio, f_m are:

$$f_0 = \dot{m}_H / \dot{m}_0^* \qquad and \qquad f_m = \dot{m}_3 / \dot{m}_2 \tag{3}$$

f_0 and f_m are related via bypass ratios, β_Q and β_R of air and fuel bypass streams; namely,

$$f_0 = \frac{(1+\beta_R)}{(1+\beta_Q)} f_m \tag{4}$$

Effective fuel-air ratio, f_e of perfectly mixed reacting stream is

$$f_e = \frac{\dot{m}_3'}{\dot{m}_2'} \tag{5}$$

Fuel and Air Mixing Efficiencies: Fuel and air mixing imperfections are characterized by specifying a fuel mixing efficiency, η_H and an air mixing efficiency, η_A. By definition η_H mass fraction of fuel mixes perfectly with η_A mass fraction of air and form perfectly mixed reacting stream. Thus,

$$\eta_H = \dot{m}_3' / \dot{m}_3 \quad \text{and} \quad \eta_A = \dot{m}_2' / \dot{m}_2 \tag{6}$$

Alternatively, fuel and air mixing imperfections are characterized by specifying an unmixed fuel bypass ratio, β_H and an unmixed air bypass ratio, β_A . These bypass ratios are related to mixing efficiencies in the following manner:

$$\beta_H = \frac{1}{\eta_H} - 1 \quad \text{and} \quad \beta_A = \frac{1}{\eta_A} - 1 \tag{7}$$

It is to be noted that effective fuel-air ratio, f_e is related to mixer fuel-air ratio, f_m via mixing efficiencies; namely

$$f_e = \frac{\eta_H}{\eta_A} \cdot f_m \tag{8}$$

Equivalence Ratio: An alternative measure for fuel-air ratio, f is equivalence ratio, ϕ which is defined as follows:

$$f = \frac{\phi \, \mathcal{M}_{H_2}}{n_{air} \, \mathcal{M}_{air}} \doteq \frac{\phi}{34.29} \tag{9}$$

where $\mathcal{M}_{H_2}$ = 2.01588 $kg/kmol$ is the molecular weight of H_2 , $\mathcal{M}_{air} = 28.9644$ $kg/kmol$ is the molecular weight of air at reference state (at 298.15K and 1 bar), and $n_{air} = 1/2\alpha$ is the number of $kmols$ of air. Here α = 0.2095 represents mole fraction of O_2 in air at reference state. Needless to say, equation (9) is proper definition of equivalence ratio provided fuel, H_2 , air, and their reaction products are insured to be at reference state. Equivalence ratio, ϕ is simply a measure and actual performance calculations always use fuel-air ratio, f .

Thrust of SCRAMJET: By extending conventional ramjet thrust expression to SCRAMJET propulsion system depicted in Figure 1, the definition of thrust T in this analysis is:

$$T = \dot{m}_6 U_6 + \dot{m}_{6'} U_{6'} + \dot{m}_{6''} U_{6''} + \dot{m}_{6'''} U_{6'''} + \dot{m}_{6^{IV}} U_{6^{IV}} - \dot{m}_0^* U_0^* + (p_6 - p_0^*) A_6 + (p_{6'} - p_0^*) A_{6'}$$
$$+ (p_{6''} - p_0^*) A_{6''} + (p_{6'''} - p_0^*) A_{6'''} + (p_{6^{IV}} - p_0^*) A_{6^{IV}} \tag{10}$$

By using definitions of mass flow rates, $\dot{m}_6 = \rho_6 U_6 A_6$, ... and of sound speeds in ideal gases, $a_6 = \sqrt{\gamma_6 p_6 / \rho_6}$ product terms $A_6 p_6$,... are expressed as follows:

$$A_6 p_6 = \dot{m}_6 U_6 \frac{1}{\gamma_6 M_6^2} \quad , \; \cdots \tag{11}$$

By referring to Figure 1 mass flow rates $\dot{m}_6$,..., $\dot{m}_{6IV}$ are expressed in term of mass flow rate, $\dot{m}_0^*$ in the following manner:

$$\left.\begin{array}{l} \dot{m}_6 = (1+f_e)\dfrac{\eta_A}{(1+\beta_Q)} \dot{m}_0^* \quad , \quad \dot{m}_{6'} = \dfrac{(1-\eta_A)}{(1+\beta_Q)} \dot{m}_0^* \quad , \quad \dot{m}_{6''} = \dfrac{(1-\eta_H)}{(1+\beta_R)} \cdot f_0 \cdot \dot{m}_0^* \\ \dot{m}_{6'''} = \dfrac{\beta_Q}{(1+\beta_Q)} \dot{m}_0^* \quad , \; and \quad \dot{m}_{6IV} = \dfrac{\beta_R}{(1+\beta_R)} \dot{m}_0^* \end{array}\right\} \tag{12}$$

When relationships in equations (11) and (12) are substituted into the equation (10), it shall be obtained that

$$\begin{aligned} \frac{T'}{\dot{m}_0^* U_0^*} = & \frac{\eta_A}{(1+\beta_Q)} \left\{ (1+f_e)\left(\frac{U_6}{U_0^*}\right)\left[1+\left(1-\frac{p_0^*}{p_6}\right)\frac{1}{\gamma_6 M_6^2}\right] - 1 \right\} + \frac{(1-\eta_A)}{(1+\beta_Q)} \left\{ \left(\frac{U_{6'}}{U_0^*}\right)\left[1+\left(1-\frac{p_0^*}{p_{6'}}\right)\frac{1}{\gamma_{6'} M_{6'}^2}\right] - 1 \right\} \\ & + \frac{(1-\eta_H)}{(1+\beta_R)} \cdot f_0 \cdot \left\{ \left(\frac{U_{6''}}{U_0^*}\right)\left[1+\left(1-\frac{p_0^*}{p_{6''}}\right)\frac{1}{\gamma_{6''} M_{6''}^2}\right] \right\} + \frac{\beta_Q}{(1+\beta_Q)} \left\{ \left(\frac{U_{6'''}}{U_0^*}\right)\left[1+\left(1-\frac{p_0^*}{p_{6'''}}\right)\frac{1}{\gamma_{6'''} M_{6''}^2}\right] - 1 \right\} \\ & + \frac{\beta_R}{(1+\beta_R)} \cdot f_0 \cdot \left\{ \left(\frac{U_{6IV}}{U_0^*}\right)\left[1+\left(1-\frac{p_0^*}{p_{6IV}}\right)\frac{1}{\gamma_{6IV} M_{6IV}^2}\right] \right\} \end{aligned} \tag{13}$$

<u>Air and Fuel Specific Impulses of SCRAMJET:</u> By definition they are

$$\text{Air specific impulse} \quad , \quad (I_{sp})_{air} = \frac{T'}{\dot{m}_0^*} \tag{14}$$

$$\text{Fuel specific impulse} \quad , \quad (I_{sp})_{fuel} = \frac{T'}{\dot{m}_H} = \frac{1}{f_0}(I_{sp})_{air} \tag{15}$$

As it is seen in equation (15) fuel specific impulse in this study is based on overall fuel consumed by SCRAMJET.

<u>Complete Enthalpy:</u> By definition complete enthalpy of a gas which is a mixture of various chemical species and in chemical equilibrium with static temperature, T and static pressure, p is

$$h \equiv h(T,p) = \frac{1}{\mathcal{M}} \sum_j{}' y_j \left[\bar{h}_f^{\,o}(T_r) + \Delta\bar{h}(T) \right]_j \tag{16}$$

where y_j is the mole fraction of j^{th} chemical species in the mixture in chemical equilibrium at T and p , $[\bar{h}_f^{\,o}(T_r)]_j$ is the molar enthalpy of formation of j^{th} chemical species at reference temperature, T_r =298.15 K and at standard pressure, p^o = 1 bar , $[\Delta\bar{h}(T)]_j$ is the molar enthalpy change of j^{th} chemical species from reference temperature, T_r to mixture temperature, T and $\mathcal{M}$ is the molecular weight of the mixture which is

$$\mathcal{M} = \sum_j y_j \mathcal{M}_j \tag{17}$$

In equation (17) $\mathcal{M}_j$ is the molecular weight of j^{th} chemical species. It is to be noted that units of complete enthalpy, h is energy per unit mass of mixture and equation (16) is structured for utilization of JANAF Thermochemical Tables (reference [2]) directly.

<u>Complete Energy</u>: The complete energy per unit mass of a gas mixture in chemical equilibrium at static temperature, T and static pressure, p with speed, U is

$$H \equiv H(T,p,U) = h(T,p) + \frac{1}{2} U^2 \tag{18}$$

<u>Entropy of Mixture</u>: Per unit mass basis entropy of a gas mixture in chemical equilibrium at static temperature, T and static pressure, p is (see for instance reference [3]):

$$s = s(T,p) = \frac{1}{\mathcal{M}} \left\{ \sum_j{}' y_j \left[\bar{s}^{\,o}(T) \right]_j - \mathcal{R} \left[\sum_j y_j \ln y_j + \ln \frac{p}{p^o} \right] \right\} \tag{19}$$

where $[\bar{s}^{\,o}(T)]_j$ is the molar entropy of j^{th} chemical species at static temperature, T and at standard pressure, p^o = 1 bar.

Ideal Process: In this analysis an ideal process is defined to be one which conserves both entropy and complete energy while system changes its thermodynamic state from k to l , that is satisfying the following two equations simultaneously.

$$\left.\begin{aligned} S(T_k, p_k) &= S(T_l, p_l) \\ H(T_k, p_k, U_k) &= H(T_l, p_l, U_l) \end{aligned}\right\} \qquad (20)$$

Stagnation Temperature and Pressure: In this analysis the concept of stagnation temperature, T_t and stagnation pressure, p_t is this: The fluid at any point of the flow field is extracted by some means with its temperature, T , pressure, p , and speed, U at the local value and then this parcel of fluid is compressed by conserving its entropy and complete energy by decreasing its speed until it vanishes. At this stagnated state its temperature and pressure is defined to be stagnation temperature, T_t and stagnation pressure, p_t . It is implied that the fluid in its original state and at stagnation state is in chemical equilibrium. Thus T_t and p_t are determined by solving following two equations simultaneously:

$$S(T_t, p_t) = S(T, p) \qquad (21)$$

$$H(T_t, p_t, U_t = 0) = H(T, p, U) \qquad (22)$$

Present method of solution uses JANAF Thermochemical Tables (reference [2]) via a table look up procedure. In these tables 6000 K is the temperature upper limit while solutions of equations (21) and (22) often shall be larger than 6000 K . For such instances, stagnation temperature and pressure calculation procedure is this: Iterative calculation procedure is structured such that the equation (21) is satisfied first by using a grid search on p_t , as a consequence of this procedure it is possible to detect last (T_t , p_t) pair say (T_n , p_n) that satisfies equation (21) and uses table look up values. At this point fluid composition is frozen with fixed values of specific heat at constant pressure, $(C_p)_n$ and specific heat ratio, γ_n and then ideal gas relations corresponding to equations (21) and (22) are used to determine stagnation temperature and pressure; namely

$$\left.\begin{aligned} H(T_n, p_n, U_n = 0) + (C_p)_n (T_t - T_n) &= H(T, p, U) \\ p_t &= p_n (T_t/T_n)^{\gamma_n/(\gamma_n - 1)} \end{aligned}\right\} \qquad (23)$$

Dynamic Pressure: By definition dynamic pressure, q is

$$q = \frac{1}{2} \rho U^2 \qquad (24)$$

or

$$q = \frac{1}{2} \gamma M^2 p \qquad (25)$$

where definition of speed of sound in an ideal gas, $a = \sqrt{\gamma p / \rho}$ was introduced into equation (24).

Specific Heat at Constant Pressure: Per unit mass basis mixture specific heat at constant pressure is

$$C_p = \frac{1}{\mathcal{M}} \sum_j{}' y_j [\bar{C}_p]_j \qquad (26)$$

where $[\bar{C}_p]_j$ is the molar specific heat at constant pressure of jth chemical species in the mixture.

MODELING OF SCRAMJET COMPONENTS

In this section modeling details of various SCRAMJET components are presented. The order of presentation is basically computational order one must follow inorder to determine fuel specific impulse of SCRAMJET from equation (15) for the propulsion system depicted in Figure 1. Details of numerical implementation of system model are beyond the scope of this report. It is sufficient to mention that almost all numerical procedures are *iterative* in nature.

Diffuser: Propulsion system station numbers 0 and 2 denote respectively inlet and exit of diffuser. The air at diffuser inlet has known static temperature, T_0 , static pressure, p_0 , and flow speed, U_0 . At this implementation T_0 is either specified or it may be set equal to the value of T_0^* . Due to lack of knowledge otherwise, it is assumed that $U_0 = U_0^*$ and $p_0 = p_0^*$. Static pressure of air at diffuser exit is assumed to be the same as that of combustor inlet, thus p_2 is a specified quantity. Diffuser model must yield static temperature, T_2 and flow speed, U_2 at diffuser exit by considering dissociation of air whenever applicable. Some details in this regard are included in the Appendix A.

Performance of an hypersonic diffuser is essentially characterized in terms of a kinetic energy efficiency, η_D which is defined as (see reference [4]):

$$\eta_D = \frac{\text{available kinetic energy after diffusion}}{\text{available kinetic energy before diffusion}} \qquad (27)$$

η_D is a specified performance parameter and according to reference [4] it is nearly constant over a wide range of values of M_0 . In the definition of η_D it is understood that the flow is ideally expanded to the same static pressure, p_0 in both numerator and denominator of equation (27). Before diffusion available kinetic energy per unit mass is $\frac{1}{2} U_0^2$. In order to determine available kinetic energy after diffusion a pseudo state $2'$ is imagined such that $p_2' = p_0$ and ideal process relationships hold; namely,

$$S(T_{2'}, p_0) = S(T_2, p_2) \qquad (28)$$

$$H(T_{2'}, p_0, U_{2'}) = H(T_2, p_2, U_2) \qquad (29)$$

Available kinetic energy after diffusion per unit mass is $\frac{U_{2'}^2}{2}$ and

$$U_{2'} = \sqrt{\eta_D}\; U_0 \qquad (30)$$

Additionally, in the diffusion process complete energy is conserved, thus

$$H(T_2, p_2, U_2) = H(T_0, p_0, U_0) \tag{31}$$

In view of equations (30) and (31), the equation (29) has only one unknown, and it can be evaluated in an iterative framework. Next, T_2 is evaluated from equation (28) again iteratively. Finally, U_2 is evaluated from equation (29) as follows:

$$U_2 = \sqrt{2\left[H(T_0, p_0, U_0) - h(T_2, p_2)\right]}$$

Liquid Fuel Source: Liquid hydrogen fuel source is characterized by specifying its static pressure, p_H and source exit flow speed, U_H . Static temperature, T_H of fuel source is assumed to be saturation temperature corresponding to pressure, p_H . T_H , p_H , and U_H are invariant with respect to amount of liquid fuel expended from the tank. Thermodynamic properties of liquid hydrogen are determined by using procedures outlined by W. C. Reynolds (reference [5]) with appropriate modifications so that property datum states and their physical units are in agreement with JANAF Thermodynamic Tables (reference [2]).

Fuel energy augmenter and heat exchangers are optional components of a SCRAMJET, consequently it is possible to inject liquid fuel from the tank directly into the combustor. In such instances, fuel bypass option request shall be ignored and fuel specific impulse contribution of unmixed fuel in the combustor shall be neglected.

Fuel Energy Augmenter: Fuel energy augmenter is an optional element of SCRAMJET. Its primary function is to control phase (liquid or gas), static temperature, T_3 , static pressure, p_3 , and speed, U_3 of fuel which is to be injected into the combustor. Such a control may be a necessity for some fuel-air mixing enhancement schemes (see Appendix D for various control options).

Fuel energy augmenter exit is identified by station $1x$ in Figures 2 and 3. In the absence of a heat exchanger this station is in fact station 3 in Figure 1.

Per unit fuel mass basis, energy input, q_f into augmenter is

$$q_f = \frac{\dot{Q}_H}{\dot{m}_H} = H(T_{1x}, p_{1x}, U_{1x}) - H(T_H, p_H, U_H) \tag{32}$$

where $\dot{Q}_H$ is also the rate at which energy is to be removed from the reactor during combustion process.

Possibility of fuel dissociation is taken into account in the treatment of fuel energy augmenter (see Appendix A for details).

Perfect Mixer: Details of the present mixer model are included in the Appendix B. Mixer exit (reactor inlet) static pressure, $p_4 = p_2$ is a specified performance parameter, consequently mixer exit static temperature, T_4 and flow speed, U_4 are determined via simultaneous solutions of complete energy and momentum equations which respectively are

$$H(T_2, p_2, U_2) + f_e \cdot H(T_3, p_3, U_3) = (1+f_e)\, H(T_4, p_4, U_4) \tag{33}$$

$$\mathcal{R}\left\{\frac{T_2}{\mathcal{M}_2 U_2} + f_e \frac{T_3}{\mathcal{M}_3 U_3}\right\} + (U_2 + f_e U_3) = \mathcal{R}_0 (1+f_e)\frac{T_4}{\mathcal{M}_4 U_4} + (1+f_e)\, U_4 \tag{34}$$

Incoming air to mixer is in chemical equilibrium at static temperature, T_2 and static pressure, p_2 with known mole fractions of species: O , N , NO , NO_2 , N_2O, O_2 , N_2 , and Ar . Similarly, fuel incoming to mixer is in chemical equilibrium at static temperature, T_3 and static pressure, p_3 with known mole fractions of species: H and H_2 . Non-reactive or mechanical mixing process is described by following "pseudo" reaction equation:

$$\phi_e (y'_H H + y'_{H_2} H_2) + (n_T)_2 (y'_O O + y'_N N + y'_{NO} NO + y'_{NO_2} NO_2 + y'_{N_2O} N_2O + y'_{O_2} O_2 + y'_{N_2} N_2 + y'_{Ar} Ar) \longrightarrow$$

$$(n_T)_4 (y_O O + y_H H + y_N N + y_{NO} NO + y_{NO_2} NO_2 + y_{N_2O} N_2O + y_{O_2} O_2 + y_{H_2} H_2 + y_{N_2} N_2 + y_{Ar} Ar) \tag{35}$$

It is to be noted that there are no new species on the right hand side of equation (39) which is characteristic of non-reactive mixing. It is implied that species on the right hand side of equation (39) are in chemical equilibrium at static temperature, T_4 and static pressure, p_4 . This is done by conserving atomic species and by conserving fuel and air masses individually. Details are presented in Appendix A.

Reactor: Details of reactor model are presented in the Appendix C. In the presence of a fuel energy augmenter, the complete energy in Appendix C is modified as follows:

$$\dot{m}_4 H(T_5, p_5, U_5) = -\dot{Q}_H + \dot{m}_4 H(T_4, p_4, U_4)$$

In view of equation (32) its alternate form is

$$H(T_5, p_5, U_5) - \left\{H(T_4, p_4, U_4) - \frac{f_e}{(1+f_e)}(1+\beta_R)(1+\beta_H)\left[H(T_{1X}, p_{1X}, U_{1X}) - H(T_H, p_H, U_H)\right]\right\} = 0 \tag{36}$$

Equation (36) and reactor momentum equation are to be solved simultaneously for T_5 and U_5 when pressure ratio across the reactor (p_5/p_4) is specified. The momentum equation for this particular case is

$$\left(\frac{U_5}{U_4}-1\right)+\frac{1}{2\gamma_4 M_4^2}\left(\frac{p_5}{p_4}-1\right)\left[\frac{T_5}{T_4}\cdot\frac{p_4}{p_5}\cdot\frac{U_4}{U_5}\cdot\frac{\mathcal{M}_{04}}{\mathcal{M}_{05}}+1\right]+\frac{C_D}{4}\left[1+\frac{T_4}{T_5}\cdot\frac{p_5}{p_4}\left(\frac{U_5}{U_4}\right)^2\frac{\mathcal{M}_{05}}{\mathcal{M}_4}\right]=0 \qquad (37)$$

When area ratio across the reactor is specified then the solution procedure need be modified as outlined in the Appendix C.

In this analysis the reactor chemical reaction is characterized by the following equation:

$$(n_T')_4(y_O'\,O+y_H'\,H+y_N'\,N+y_{NO}'\,NO+y_{NO_2}'\,NO_2+y_{N_2O}'\,N_2O+y_{O_2}'\,O_2+y_{H_2}'\,H_2+y_{N_2}'\,N_2+y_{Ar}'\,Ar)\longrightarrow$$

$$(n_T)_5\,(y_O\,O+y_H\,H+y_{OH}\,OH+y_{H_2O}\,H_2O+y_N\,N+y_{NO}\,NO+y_{HO_2}\,HO_2+y_{NO_2}\,NO_2$$

$$+\,y_{N_2O}\,N_2O+y_{O_2}\,O_2+y_{H_2}\,H_2+y_{N_2}\,N_2+y_{Ar}\,Ar\,) \qquad (38)$$

Combustion products are assumed to be in chemical equilibrium at static temperature, T_5 and static pressure p_5 . Details of mole fraction calculation procedures of this case are also available in the Appendix A.

<u>Nozzle(s)</u>: All nozzles depicted in Figure 1 are characterized in the manner as outlined below: Nozzle exit is identified by station 6 and it is assumed that nozzle exhaust pressure, p_6 or nozzle expansion coefficient, $C_e = p_6/p_0^*$ is specified (C_e = 1 for an ideally expanded nozzle, C_e < 0 for an over expanded nozzle and C_e > 0 for an under expanded nozzle).

Conservation of complete energy is

$$H(T_6\,,p_6\,,U_6)=H(T_5\,,p_5\,,U_5) \qquad (39)$$

Nozzle flow viscous effects are taken into account by specifying a nozzle velocity coefficient, C_v which is defined as (see reference [4]):

$$C_U = \frac{\text{actual exhaust velocity, } U_6}{\text{ideal exhaust velocity, } U_6^*} \tag{40}$$

By definition ideal exhaust velocity, U_6^* is the velocity associated with an ideal process in which nozzle exhaust pressure is p_6 ; namely U_6^* is obtained by solving simultaneously following equations:

$$\left.\begin{aligned} s(T_6^*, p_6) &= s(T_5, p_5) \\ h(T_6^*, p_6) + \tfrac{1}{2} U_6^{*2} &= H(T_5, p_5, U_5) \end{aligned}\right\} \tag{41}$$

Next, by using $U_6 = C_U U_6^*$ and equation (39) nozzle exhaust static temperature, T_6 is determined. For an ideal nozzle, $C_U = 1.0$.

It is implied that all nozzle computation procedures take into account recombination of chemical species as detailed in the Appendix A. That is to say when the nozzle flow is equilibrium composition flow, then mole fractions of species corresponding to chemical equilibrium are determined at appropriate static temperature and pressure. On the other hand when the nozzle flow is frozen composition flow then mole fractions of species at nozzle inlet are used throughout nozzle exit state determination procedures.

When it is desired to determine the state of flow at a particular location within the nozzle identified by flow static pressure, p_i then outlined computation procedure is modified slightly. A convenient approach is to visualize N nozzle stations along the nozzle axis. Designate station 0 to be the nozzle inlet then station N identifies the nozzle exit. Let π to be the ratio of two consecutive station pressures; namely

$$\frac{p_{i+1}}{p_i} = \pi \qquad for \quad i = 0, 1, \dots, N-1 \tag{42}$$

For this choice, value of π is

$$\pi = (p_6 / p_5)^{1/N} \tag{43}$$

The nozzle velocity coefficient, C_U is a characteristic value as a unit. It is reasonable to assign nozzle velocity coefficient, $(C_U)_i$ for the ith nozzle station in the following form:

$$(C_U)_i = (p_i / p_5)^{\beta} \qquad for \quad i = 0, 1, \dots, N \tag{44}$$

where

$$\beta = \frac{\ln C_U}{\ln (p_6/p_5)} \tag{45}$$

It is to be noted that $(C_U)_0 = 1$ and $(C_U)_N = C_U$.

With this computation procedure, nozzle flow composition may be frozen at a desired nozzle station identified by the flow pressure.

When ith nozzle station state is determined then area ratios, A_i/A_5 are

$$\frac{A_i}{A_5} = \frac{T_i}{T_5} \cdot \frac{p_5}{p_i} \cdot \frac{U_5}{U_i} \cdot \frac{\mathcal{M}_{b5}}{\mathcal{M}_{bi}} \qquad for\ i = 1, \ldots, N \tag{46}$$

Heat Exchanger at Diffuser Exit: Heat exchanger at diffuser exit is shown in Figure 2 and it is an optional element of SCRAMJET. Whenever necessary it may be used to control static temperature of air after diffusion process. Stations 1x and 2x designate respectively fuel energy augmenter and diffuser exits. As implied in Figure 2, heat exchanging air and fuel streams are not in physical contact.

Complete energy equation for heat exchanger is

$$H(T_2, p_2, U_2) - \left\{ H(T_{2x}, p_{2x}, U_{2x}) + f_x \left[H(T_{1x}, p_{1x}, U_{1x}) - H(T_1, p_1, U_1) \right] \right\} = 0 \tag{47}$$

where

$$f_x = \frac{\dot{m}_H}{\dot{m}_\ell} = (1+\beta_\ell) f_m$$

In this study it is assumed that heat exchanger pressure drops are negligible, thus $p_1 = p_{1x}$ and $p_2 = p_{2x}$ and there are no flow frictional losses, thus $U_1 = U_{1x}$ and $U_2 = U_{2x}$. With these assumptions equation (47) still contains two unknowns; namely, T_1 and T_2 . Consequently, one of these unknowns must be specified and then equation (47) is to be used to determine the other one. Presently, static temperature of fuel at heat exchanger exit, T_1 is considered to be a specified quantity.

A parameter, η_x is introduced to measure effectiveness of heat exchanger:

$$\eta_x = \frac{H(T_1, p_1, U_1) - H(T_{1x}, p_{1x}, U_{1x})}{\frac{1}{f_x} H(T_{2x}, p_{2x}, U_{2x}) - H(T_{1x}, p_{1x}, U_{1x})} \tag{48}$$

When $\eta_x = 0$ then complete energies of fuel and air streams are conserved individually and $T_1 = T_{1x}$ and $T_2 = T_{2x}$. On the otherhand, when $\eta_x = 1$ then complete energies of fuel and air streams are exchanged. In situations that one wishes to specify heat exchanger effectiveness parameter, η_x then equations (47) and (48) are solved simultaneously for T_1 and T_2 .

Heat Exchanger at Reactor Exit: Heat exchanger at reactor exit is shown in Figure 3 and it is an optional element of SCRAMJET. In contrast with fuel energy augmenter which increases both kinetic energy and sensible heat energy of fuel, heat exchangers are used to increase only sensible heat of fuel.

Characterization of this heat exchanger is similar to heat exchanger at diffuser exit. Complete energy equation is

$$H(T_5, p_5, U_5) - \left\{ H(T_{5x}, p_{5x}, U_{5x}) + f_x \left[H(T_{1x}, p_{1x}, U_{1x}) - H(T_1, p_1, U_1) \right] \right\} = 0 \tag{49}$$

where

$$f_x = \frac{\dot{m}_H}{\dot{m}_5} = \frac{(1+\beta_R) f_m}{\eta_H f_m + \eta_A}$$

and heat exchanger effectiveness parameter, η_x in this case is

$$\eta_x = \frac{H(T_1, p_1, U_1) - H(T_{1x}, p_{1x}, U_{1x})}{\frac{1}{f_x} H(T_{5x}, p_{5x}, U_{5x}) - H(T_{1x}, p_{1x}, U_{1x})} \tag{50}$$

Equation (49) is used to determine, T_5 with specified value of T_1 and with assumptions $p_1 = p_{1x}$, $p_5 = p_{5x}$, $U_1 = U_{1x}$, and $U_5 = U_{5x}$.

By using preceding SCRAMJET component models a FORTRAN program was written in order to predict performance of SCRAMJET propulsion systems depicted in Figures 1, 2, and 3. This program uses a number of somewhat involved numerical iterative procedures. Their descriptions are beyond the scope of this report.

Appendix D includes some clarifications of program message that users of this program may find of interest.

AN EXPLORATION OF SCRAMJET PARAMETER VARIATION EFFECTS

SCRAMJET performance analysis model includes a number of parameters and their values need be set by designers. The intent in this section is to highlight effects of some parameter variations on fuel specific impulse developed by propulsion system.

Parameters which are at the disposal of a designer are best described by the characterization of what is referred to as IDEAL SCRAMJET in this report:

1. Flight path: $z_0^* = 5.526\ (M_0^* + 10.286)$, kft (See Figure 4 and it is referred to as orbital flight path)
2. Diffuser:
   - 2.1 Diffuser inlet air static temperature, $T_0 = T_0^*$.
   - 2.2 Diffuser exit static pressure, $p_2 = p_4 = 1\ atm$.
   - 2.3 Diffuser kinetic energy efficiency, $\eta_D = 1.0$.
3. Liquid fuel tank exit:
   - 3.1 Static pressure, $p_H = 1\ atm$.
   - 3.2 Speed, $U_H = 0.0$.
4. Fuel energy augmenter exit:
   - 4.1 Phase = gas
   - 4.2 Static Temperature, $T_{1X} = 491.67\ °R\ (273.15\ K)$.
   - 4.3 Static Pressure, $p_{1X} = 1\ atm$.
   - 4.4 Speed, $U_{1X} = U_2$.
5. Heat exchangers: none
6. Mixer:
   - 6.1 Fuel-air mixing is perfect ($\eta_A = 1.0$ and $\eta_H = 1.0$)
   - 6.2 Fuel-air mass ratio, $f_m = f_0 = 0.035$ ($\phi_m = \phi_0 = 1.2$)
7. Reactor:
   - 7.1 Constant pressure type, $p_4 = p_5 = 1\ atm$.
   - 7.2 Drag coefficient, $C_D = 0.0$.
8. Nozzles:
   - 8.1 Ideally expanded, $C_e = p_e / p_0^* = 1.0$.
   - 8.2 Nozzle velocity coefficient, $C_U = 1.0$.
   - 8.3 Equilibrium composition flow
9. Air and fuel bypasses: none ($\beta_a = 0$ and $\beta_R = 0$)

A number of parameter variations to be considered below are in essence deviations from IDEAL SCRAMJET propulsion system.

Some results to be presented shall refer to a REAL SCRAMJET. It is an IDEAL SCRAMJET except:

2.3 Diffuser kinetic energy efficiency, η_D = 0.985
8.2 Nozzle velocity coefficient, C_V = 0.985

Real Gas Effects: Present model captures essential characteristics of real gas effects by allowing chemical species to dissociate in diffusion and combustion processes and then to recombine in nozzle expansion process optionally. Dissociation and recombination of chemical species always obey chemical equilibrium law of mass action. Equilibrium and frozen composition nozzle flows refer to complete recombination and no recombination of chemical species respectively. In examples below, partial recombination of chemical species are also considered by freezing composition of gas at nozzle pressures, $P_{fr} = 1/64$ atm. and $1/512$ atm.

Real gas effects are compared with results of analyses in which no dissociation of chemical species are allowed in diffusion and combustion processes and nozzle flow is frozen composition type. In particular, analyses consider H_2 is to be the fuel and air chemical species are O_2, N_2, and A_r with mole fractions $y'_{O_2} = 0.2095$, $y'_{N_2} = 0.7809$, and $y'_{Ar} = 0.0096$ respectively. Following reactions are assumed to describe combustion process:

when $\phi_e \leq 1$:

$$\phi_e H_2 + \frac{1}{2}\left(O_2 + \frac{y'_{N_2}}{y'_{O_2}} N_2 + \frac{y'_{Ar}}{y'_{O_2}} Ar\right) \rightarrow \phi_e H_2O + \frac{1}{2}(1-\phi_e)O_2 + \frac{1}{2}\left(\frac{y'_{N_2}}{y'_{O_2}} N_2 + \frac{y'_{Ar}}{y'_{O_2}} Ar\right)$$

and when $\phi_e \geq 1$, :

$$\phi_e H_2 + \frac{1}{2}\left(O_2 + \frac{y'_{N_2}}{y'_{O_2}} N_2 + \frac{y'_{Ar}}{y'_{O_2}} Ar\right) \rightarrow H_2O + (\phi_e - 1) H_2 + \frac{1}{2}\left(\frac{y'_{N_2}}{y'_{O_2}} N_2 + \frac{y'_{Ar}}{y'_{O_2}} Ar\right)$$

For an IDEAL SCRAMJET results are presented in Figure 5. Behaviors associated with particular partial recombination cases and no dissociation case are not distinguishable from equilibrium composition nozzle flow behavior. This is expected because IDEAL SCRAMJET operation temperatures are not high enough to cause significant dissociation.

To accentuate real gas effect a REAL* SCRAMJET is considered which differs from IDEAL SCRAMJET as follows:

2.3 Diffuser kinetic energy efficiency, η_D = 0.985

5. Heat exchanger at reactor exit and fuel temperature, $T_f = 3000\ ^\circ R$.

6.1 Air and fuel mixing efficiencies respectively are
$\eta_A = 0.75$ and $\eta_\mu = 0.75$.

7.1 Reactor drag coefficient, $C_b = 0.10$.

8.1 Nozzle velocity coefficient, $C_v = 0.985$.

REAL* SCRAMJET results are presented in Figures 6 (a) and (b). It is to be noted that predictions of fuel specific impulse (Figure 6 (b)) and temperatures at diffuser exit (Figure 7(a)) and at reactor exit (Figure 7 (b)) by using analyses based on no-dissociation of chemical species may be erroneous.

Nozzle Expansion Coefficient, C_e Effects: By definition nozzle expansion coefficient, $C_e = p_e / p_0^*$, where p_e is nozzle exhaust static pressure and p_0^* is free-stream static pressure. For an ideally expanded nozzle, $C_e = 1$, and $C_e > 1$ and $C_e < 1$, characterize respectively under and over expanded nozzles. Performance of a SCRAMJET which has either under or over expanded nozzle is inferior to that of IDEAL SCRAMJET as illustrated in Figure 8. For the present flight path nozzle expansion ratio, $\varepsilon = p_5 / p_e = p_5 / (p_0^* C_e)$ increases exponentially with flight Mach number, M_0^* as shown in Figure 9.

By increasing nozzle expansion coefficient, C_e as flight Mach number, M_0^* increases, it is possible to maintain expansion ratio, ε at a constant value. Computation results pertaining this case are presented in Figure 10. It is to be noted that Figure 10 illustrates the fact that IDEAL SCRAMJET performance is the optimum.

Figure 11 illustrates performance characteristics of SCRAMJET at various constant altitude flight paths. An increase in altitude, z_0^* is in effect an increase in nozzle expansion ratio, ε. Similarly, for a constant pressure combustor, an increase in combustor pressure is in effect an increase in nozzle expansion ratio as shown in Figure 12.

Combustor Type Effects: Effects of increasing pressure ratio, p_5/p_4 or decreasing area ratio, A_5/A_4 across the combustor are to increase SCRAMJET fuel specific impulse slightly at low flight Mach numbers. Either situation is analogous to increasing nozzle expansion ratio, ε. Results of such calculations are not included in this report.

As compared to IDEAL SCRAMJET, a constant cross-sectional area combustor SCRAMJET performs slightly better at low flight Mach numbers as shown in Figure 13.

Figure 14 presents pressure ratio and cross-sectional area ratio variations on the present flight path respectively for a constant cross-sectional area combustor SCRAMJET and for IDEAL SCRAMJET.

<u>Heat Exchanger Effects:</u> IDEAL SCRAMJET performance is not altered by the presence of a heat exchanger either at diffuser exit or at reactor exit. Heat exchangers simply redistribute heat energy within the propulsion system.

<u>Air and Fuel Bypass Effects:</u> In view of equation (13), any amount of air or fuel bypass introduction into the propulsion system of an otherwise IDEAL SCRAMJET shall result in performance degradation as illustrated in Figures 15 and 16. In order to insure that fuel remains in gaseous phase during its expansion through the nozzle, it was necessary to activate heat exchanger at reactor exit with $T_{5IV} = T_1 = 3000\,^{\circ}K$.

<u>Mixing Inefficiency Effects:</u> Figure 17 illustrates degradation of SCRAMJET performance as a result of inefficient air and fuel mixing. It is to be noted that present model of air and fuel mixing inefficiencies is in effect compulsory air and fuel bypass introduction into the propulsion system.

<u>Equivalence Ratio Effects</u>: Figure 18 presents temperature increase ($T_5 - T_4$) due to combustion as a function of effective equivalence ratio, ϕ_e for flight Mach numbers, M_0^* = 6, 15, and 25. At high flight Mach numbers, significant dissociation of combustion products results in lower temperature increase. Maximum temperature increase occurs in the neighborhood of 1.1 $< \phi_e <$ 1.2 and for this reason $\phi_e = 1.2$ value was assigned to IDEAL SCRAMJET.

Influence of effective equivalence ratio on SCRAMJET fuel specific impulse is illustrated in Figures 19 (a) and (b). These figures reflect the fact that fuel specific impulse is inversely proportional to effective equivalence ratio.

<u>Effect of Component Losses</u>: Figures 20 (a), (b), and (c) illustrate degradation of fuel specific impulse of a SCRAMJET due to respectively losses in diffuser, reactor and nozzle. Needless to say these components must be designed carefully.

PRELIMINARY RESULTS FROM PERFORMANCE PREDICTION MODEL

Unless otherwise stated, all results which are to be presented in this section refer to a REAL SCRAMJET. As it was mentioned in the previous section a REAL SCRAMJET is an IDEAL SCRAMJET except its diffuser kinetic energy efficiency, $\eta_D = 0.985$ and its nozzle velocity coefficient, $C_u = 0.985$

Constant Dynamic Pressure Flight Path Behavior of SCRAMJET:

In order to maintain in flight structural integrity of a SCRAMJET, it must not be subjected to dynamic pressures greater than a nominal value (in this report it is speculated to be 0.5 atm). For constant dynamic pressure flight paths depicted in Figure 21, fuel specific impulse of a SCRAMJET does not vary significantly as illustrated in Figure 22. At a specified flight Mach number, lower dynamic pressure implies higher flight altitude or higher nozzle expansion ratio, thus higher fuel specific impulse. Specific impulse variation with dynamic pressure does not appear to be significant for flight Mach numbers above 15.

A Simulation of Film Cooling Effects:

With present SCRAMJET performance prediction model a primitive simulation of effects of film cooling of combustor by using fuel may be brought into the light. It is expected that amount of fuel to be used for film cooling shall increase with flight Mach number. This can be simulated by specifying overall equivalence ratio, ϕ_0 as an increasing function of M_0^* while keeping effective equivalence ratio at a fixed value, $\phi_e = 1.2$. Present treatment uses a linear relationship such as illustrated in Figure 23 (a). Assuming that air mixing efficiency is perfect, this results in variation of fuel mixing efficiency, η_M with flight Mach number, M_0^* as depicted in Figure 23 (b).

Heat transfer process due to film cooling is simulated by activating the heat exchanger at reactor exit (See Figure 3) with fuel exit temperature is set at $T_1 = 3000\ ^oR$.

Comparative variations of fuel specific impulse with flight Mach number are illustrated in Figure 24 when SCRAMJET is on proposed orbital flight path, that is $z_0^* = 5.526\ (M_0^* + 10.286\ )$ and Figure 25 presents similar results where SCRAMJET is on $q_0^* = 0.5\ atm.$ dynamic pressure flight path. In all computations, unmixed fuel nozzle expansion is carried out by obeying equilibrium composition flow rules.

Film cooling always degrades SCRAMJET fuel specific impulse when perfectly mixed stream nozzle expansion obeys equilibrium composition flow rules. On the other hand film cooling improves specific impulse at high flight Mach numbers when perfectly mixed stream nozzle expansion obeys frozen composition flow rules.

Effectiveness of heat exchanger, η_x as given in equation (50), is degraded as flight Mach number is increased. This is illustrated in Figure 26.

Constant Combustor Inlet Mach Number Flight Path Behavior of SCRAMJET

To achieve a good combustion efficiency, it may be necessary to operate SCRAMJET combustor at constant inlet Mach number at all flight conditions. Flight paths of a SCRAMJET at various combustor inlet Mach numbers are illustrated in Figure 27 and compared with proposed orbital and constant dynamic pressure flight paths.

Figures 28 (a) through (g) present double parametric study results in order to shed some insight to the particular performance behavior of a SCRAMJET. In each figure reactor inlet Mach number, M_4 is a parameter and from one figure to the next composition freezing pressure in the nozzle decreases. Figures 28 (a) and (g) respectively correspond to frozen and equilibrium composition nozzle flows.

When Figure 28 (a) and (g) are superposed then it shall be observed that branches of curves to the left of peaks are almost the same, that is frozen and equilibrium composition nozzle flows yield almost identical performances. This is expected since operation of SCRAMJET on left branch correspond to very low flight altitudes (see Figure 27) at which composition of combustion products freeze immediately. When nozzle flow composition is frozen either completely or partially such as shown in Figure 28 (a) through (f), then for a specified flight Mach number the maximum value of fuel specific impulse occurs at a definite value of reactor inlet Mach number. As a consequence SCRAMJET performance can be optimized. for example, dashed performance curve which envelops right branches of performance curves is the optimum performance curve for completely frozen composition nozzle flow. Figure 29 presents accurately determined optimum performance curves of this case as well as for a typical partially frozen composition nozzle flow. The latter case exhibits a slope discontinuity in the vicinity of flight Mach number, $M_0^* = 21.5$. This particular slope discontinuity is a jump in reactor inlet Mach number value as depicted in Figure 30 which is presentation of optimum performance curve as a function of reactor inlet Mach number. An alternative presentation of this jump phenomenon is found in Figure 31. This report offers no explanation of this phenomenon.

Prediction of Dynamic Performance of SCRAMJET (See reference [13] for details)

Assuming that SCRAMJET is in a planar flight trajectory, the component of its equation of motion tangent to flight path is

$$m(t)\frac{dU_0^*(t)}{dt} = T(t) - D(t) - m(t)g\frac{1}{U_0^*(t)}\cdot\frac{dz(t)}{dt} \tag{51}$$

Thrust, T (t) is given by equation (15); namely $T(t) = f_0 \cdot (\rho_0^* U_0^* A_0^*)(I_{sp})_{fuel}$.

In this treatment, conventional vehicle drag law is used, that is

$$D(t) = \frac{1}{2} C_f A_D \rho_0^* U_0^{*2}$$

For turbulent of flow, the expression for skin friction, Cf is given by von Kármán [8] is used (See for instance reference [11]):

$$C_f = \frac{0.455}{[\log_{10} Re]^{2.58}}$$

and for laminar flow, the expression for skin friction, Cf given by Blasius [9] is used (see for instance reference [12]):

$$C_f = \frac{1.328}{\sqrt{Re}}$$

Absolute viscosity of air, μ_0^* which enters into evaluation of Reynold's number, $Re = (\rho_0^* U_0^* L)/\mu_0^*$, is determined by using the empirical formula given in reference [1]; namely,

$$\mu_0^* = \frac{(1.458\times10^{-6})\, T_0^{*\,3/2}}{(T_0^* + 110.4)}$$

where μ_0^* is in SI units.

Previous fuel specific impulse determination procedures are slightly modified so that primary independent variables are flight speed, U_0^* and flight altitude, $z(t)$ and then equation (51) is solved numerically by using a fourth order predictor-corrector method which starts with the minimum error bound and fourth order Runge-Kutta method.

Exploratory calculations are based on a hypothetical vehicle whose geometry is depicted in Figure 32 and whose essential particulars are:

(i) Total (structural+payload+fuel) initial mass of vehicle,

$$m(t=0) = 125{,}000 \ kg \quad (275{,}000 \ lbm).$$

(ii) Initial mass of fuel = 45,000 kg (99,000 lbm)

(iii) Vehicle capture area, $A_0^* = 49 \ m^2$ (527 ft^2).

(iv) Vehicle characteristic drag to capture area ratio, $A_D/A_0^* = 38$

(v) Vehicle length, $\ell = 60 \ m$ (197 ft).

Figures (33) and (34) present dynamic simulation results where hypothetical vehicle is on 0.5 atm constant dynamic pressure flight trajectory and uses respectively propulsion system which are referred to as REAL and IDEAL SCRAMJETS in this report. For all simulations initial vehicle speed, $U_0^*(t=0) = 2133.6 \ m/s$ which correspond to flight Mach number, $M_0^* = 7.1$ at altitude $Z_0^* = 92.8 \ kft$.

These figures clearly indicate that present hypothetical vehicle may reach orbital speed when propulsion system components are nearly perfect and vehicle skin friction corresponds to laminar flow.

Additional calculations also indicate that there exists an optimum flight trajectory such that when amount of fuel to be expended in a given time interval is specified, then the vehicle attains the maximum speed at a definite altitude. However, on this optimum trajectory vehicle is subject to dynamic pressures which are significantly larger than nominal 0.5 atm.

CONCLUDING REMARKS

A FORTRAN program was developed in order to determine the effects of design choices on the SCRAMJET fuel specific impulse in the context of real gas effects. With the aid of this program, a systematic parametric study in the form of as deviations from the IDEAL SCRAMJET performance was carried out. Results indicate that fuel specific impulse of the SCRAMJET is very sensitive to diffuser, combustor, nozzle and air-fuel mixing inefficiencies. When chemical composition of flow through the nozzle is either partially or fully frozen then fuel specific impulse of the SCRAMJET can be optimized with respect to combustor inlet Mach number. A simple minded simulation of combustor film cooling process shows that the SCRAMJET fuel specific impulse will improve when chemical composition of flow through the nozzle has a tendency to freeze.

Another FORTRAN program was developed in order to determine dynamic performance of the variable mass SCRAMJET. This program uses previous program as a subroutine. Results indicate that a hypothetical vehicle may attain orbital speed when components of the SCRAMJET are nearly perfect and vehicle skin friction corresponds to laminar flow.

REFERENCES

[1] U.S. Standard Atmosphere, 1976, National Oceanic and Atmospheric Administration, NACA, U.S. Air Force, Washington, D. C., October 1976.

[2] JANAF Thermochemical Tables, Journal of Physical and Chemical Reference Data, Volume 14, 1985, Supplement No. 1, the American Chemical Society and American Institute of Physics for the National Bureau of Standards.

[3] G. J. Van Wylen and R. E. Sonntag, "Fundamentals of Classical Thermodynamics", John Wiley and Sons, 1978 (2nd edition).

[4] J. L. Kerrebrock, "Aircraft Engines and Gas Turbines", The MIT Press, 1983 (ch 10).

[5] W. C. Reynolds, "Thermodynamic Properties in SI", Stanford University Press, 1979.

[6] A. M. Kuethe and C. Y. Chow, "Foundations of Aerodynamics: Bases of Aerodynamic Design", John Wiley and Sons, 1979 (3rd edition, pp. 213-214).

[7] J. V. Becker and D. D. Baals, "Analysis of Heat and Compressibility Effects in Internal Flow Systems and High Speed Tests of a Ramjet System", NACA Report No. 773, 1943.

[8] Theodore von Kármán, "Turbulence and Skin Friction", J. Aeronautical Sci., Vol. 1, No. 1, 1934.

[9] H. Blasius, "Grenzschichten in Flüssigkeinten mit kleiner Reibung", Z. math. und Physik, Vol. 56, 1908.

[10] "Hypersonic Technology for Military Application", Committee on Hypersonic Technology for Military application Air Force Studies Board Commission on Engineering and Technical Systems National Research Council, National Academic Press, Washington, D.C., 1989.

[11] R. H. Sabersky and A. J. Acosta, "Fluid Flow A First Course in Fluid Mechanics", The Macmillan Co., New York, 1964 (p. 255).

[12] H. Rouse, "Fluid Mechanics for Hydraulic Engineers", McGraw-Hill Book Co., New York, 1938 (p. 200).

[13] S. Serdengecti, "A Note on the Prediction of Variable Mass Vehicle Dynamics of SCRAMJET", Engineering Department Report, Harvey Mudd College, Claremont, CA 91711, March 28, 1991.

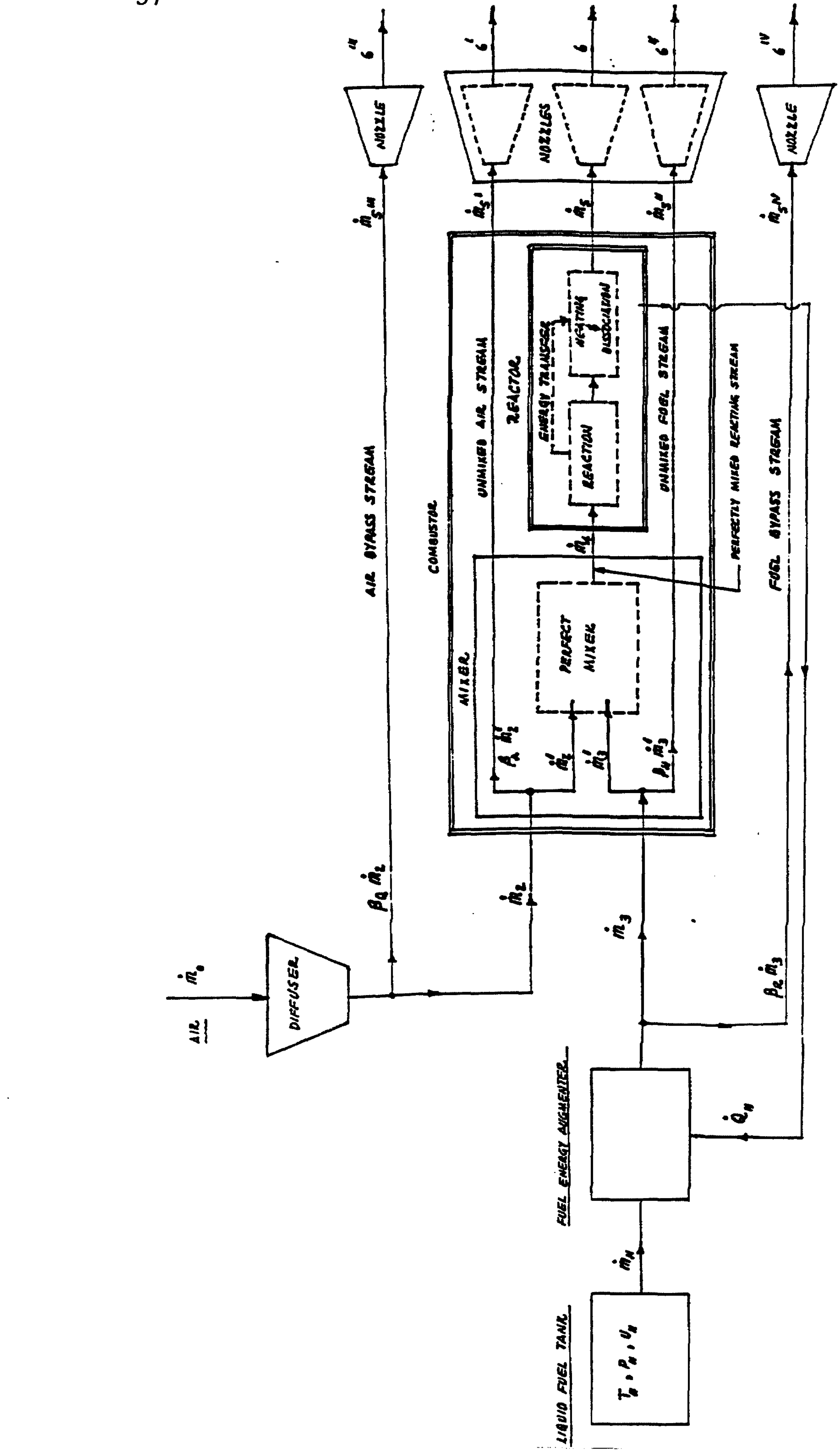

AIR
DIFFUSER
LIQUID FUEL TANK
FUEL ENERGY AUGMENTER
AIR BYPASS STREAM
COMBUSTOR
MIXER
PERFECT MIXER
REACTOR
REACTION
ENERGY TRANSFER
HEATING
DISSOCIATION
UNMIXED AIR STREAM
UNMIXED FUEL STREAM
PERFECTLY MIXED REACTING STREAM
FUEL BYPASS STREAM
NOZZLE
NOZZLES
NOZZLE

Figure 1

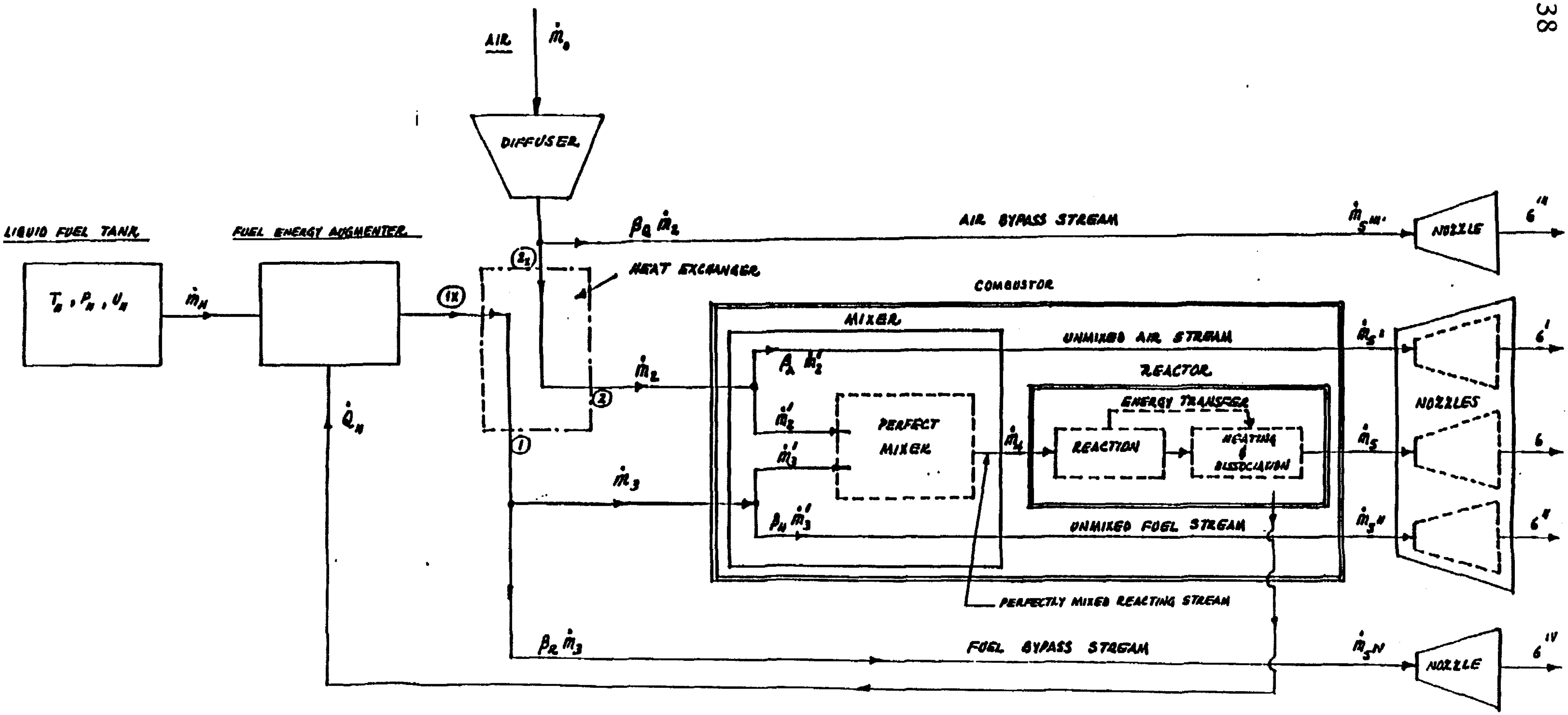

AIR
$\dot{m}_0$
DIFFUSER
LIQUID FUEL TANK
T_H, P_H, U_H
$\dot{m}_H$
FUEL ENERGY AUGMENTER
$\dot{Q}_H$
(1X)
(2₁)
HEAT EXCHANGER
(2)
(1)
$\beta_a \dot{m}_2$
AIR BYPASS STREAM
$\dot{m}_2$
$\dot{m}_3$
COMBUSTOR
MIXER
$\beta_\lambda \dot{m}_2'$
$\dot{m}_2'$
$\dot{m}_3'$
$\beta_u \dot{m}_3'$
PERFECT MIXER
$\dot{m}_4$
UNMIXED AIR STREAM
REACTOR
ENERGY TRANSFER
REACTION
HEATING & DISSOCIATION
UNMIXED FUEL STREAM
PERFECTLY MIXED REACTING STREAM
$\beta_R \dot{m}_3$
FUEL BYPASS STREAM
NOZZLE
NOZZLES
$6''$
$6'$
6
$6''$
6^{IV}

Figure 2

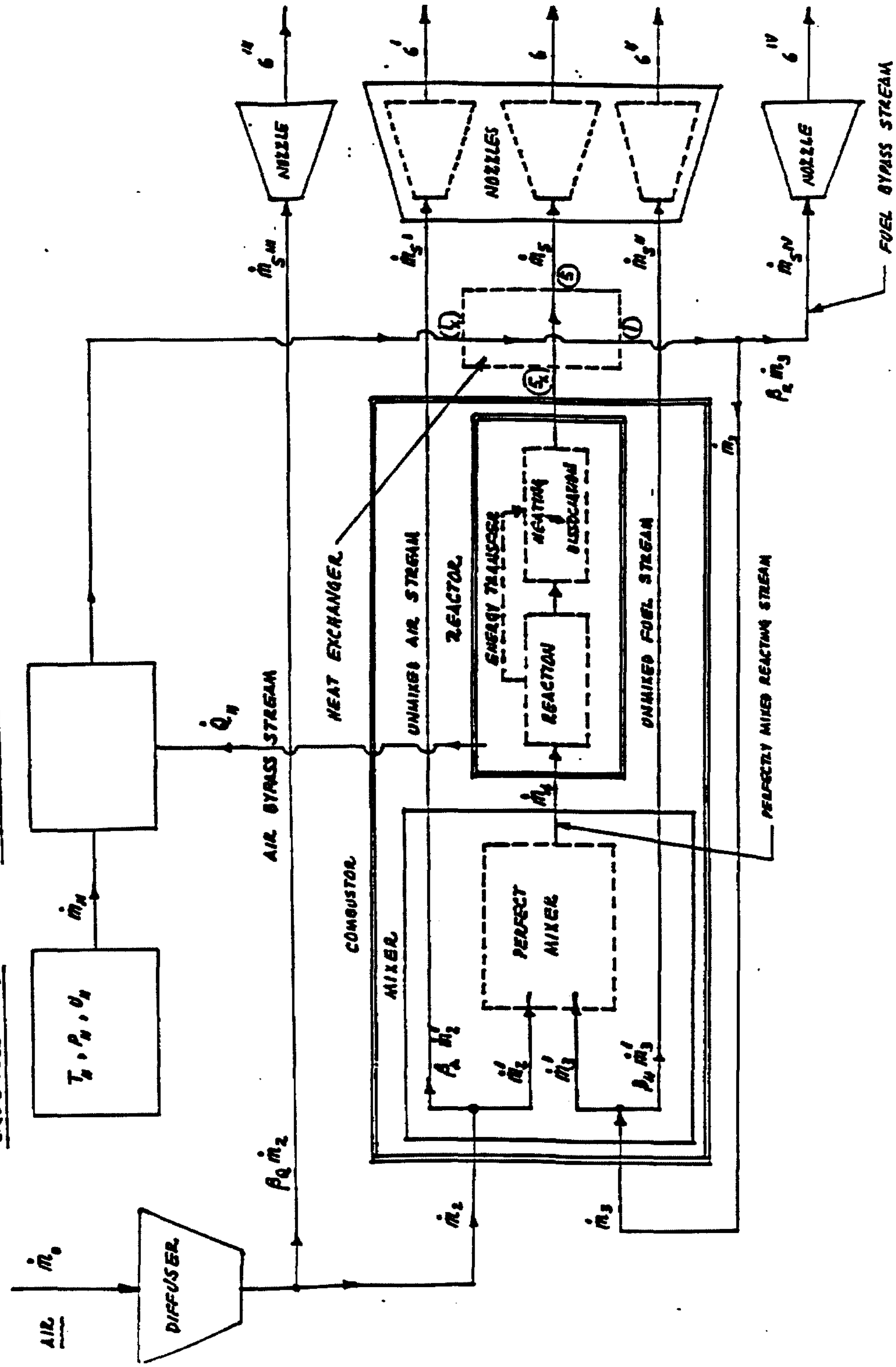

AIR
DIFFUSER
LIQUID FUEL TANK
FUEL ENERGY AUGMENTER
AIR BYPASS STREAM
COMBUSTOR
MIXER
PERFECT MIXER
HEAT EXCHANGER
UNMIXED AIR STREAM
REACTOR
ENERGY TRANSFER
REACTION
HEATING & DISSOCIATION
UNMIXED FUEL STREAM
PERFECTLY MIXED REACTING STREAM
NOZZLE
NOZZLES
NOZZLE
FUEL BYPASS STREAM

Figure 3

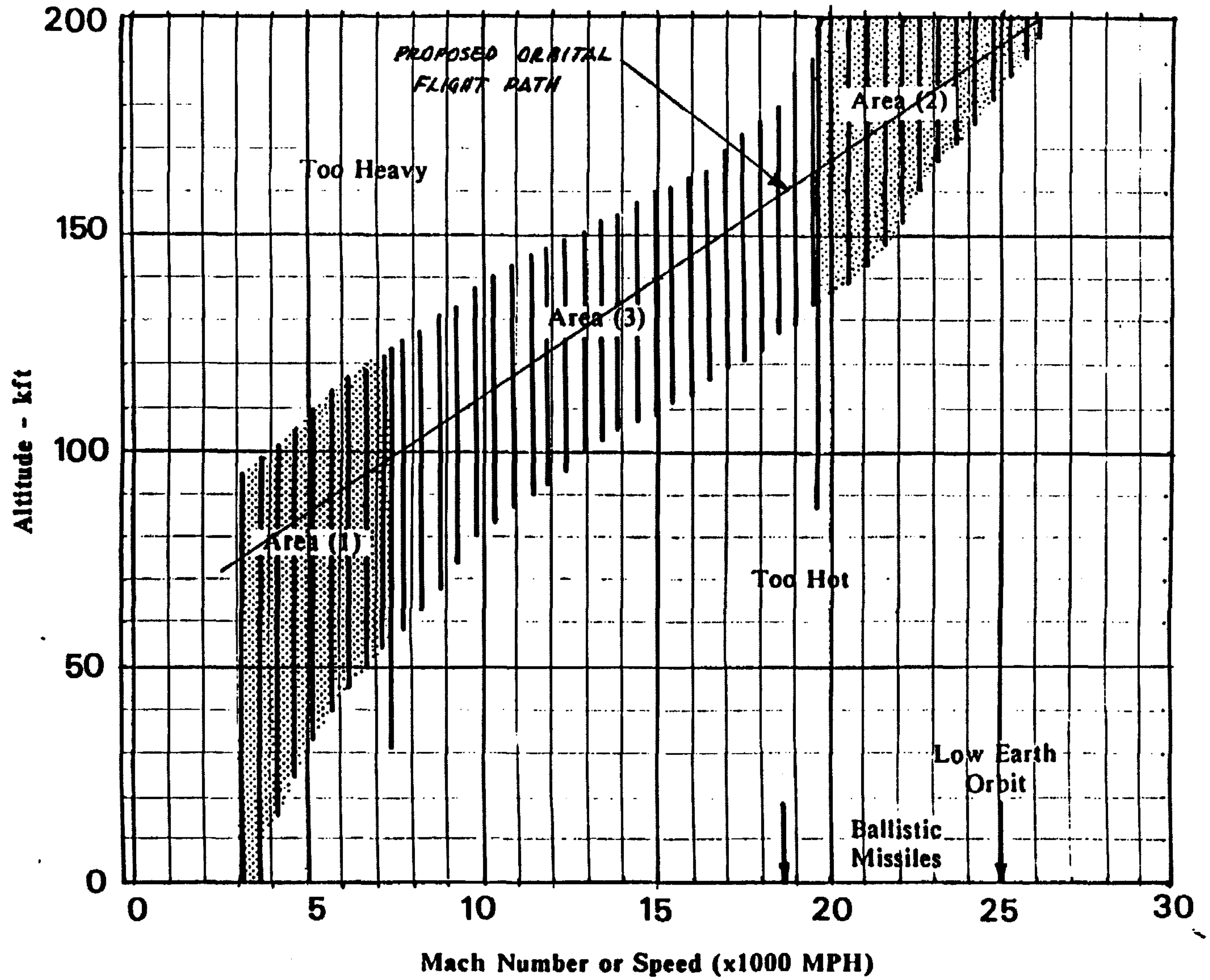


Figure 4

FIGURE 1-2

Approximate Corridor of Steady (Cruising) Flight

(from reference [10])

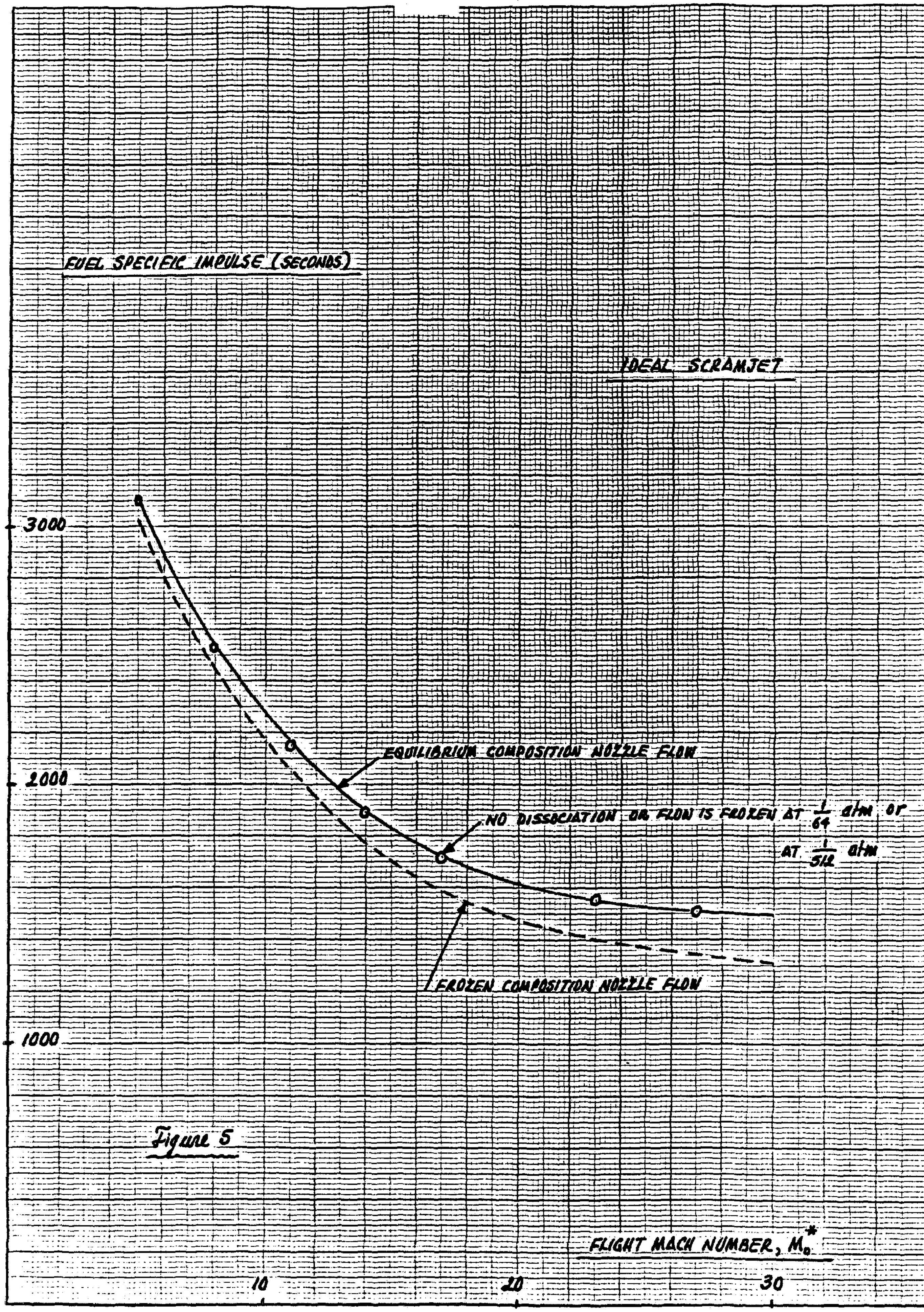
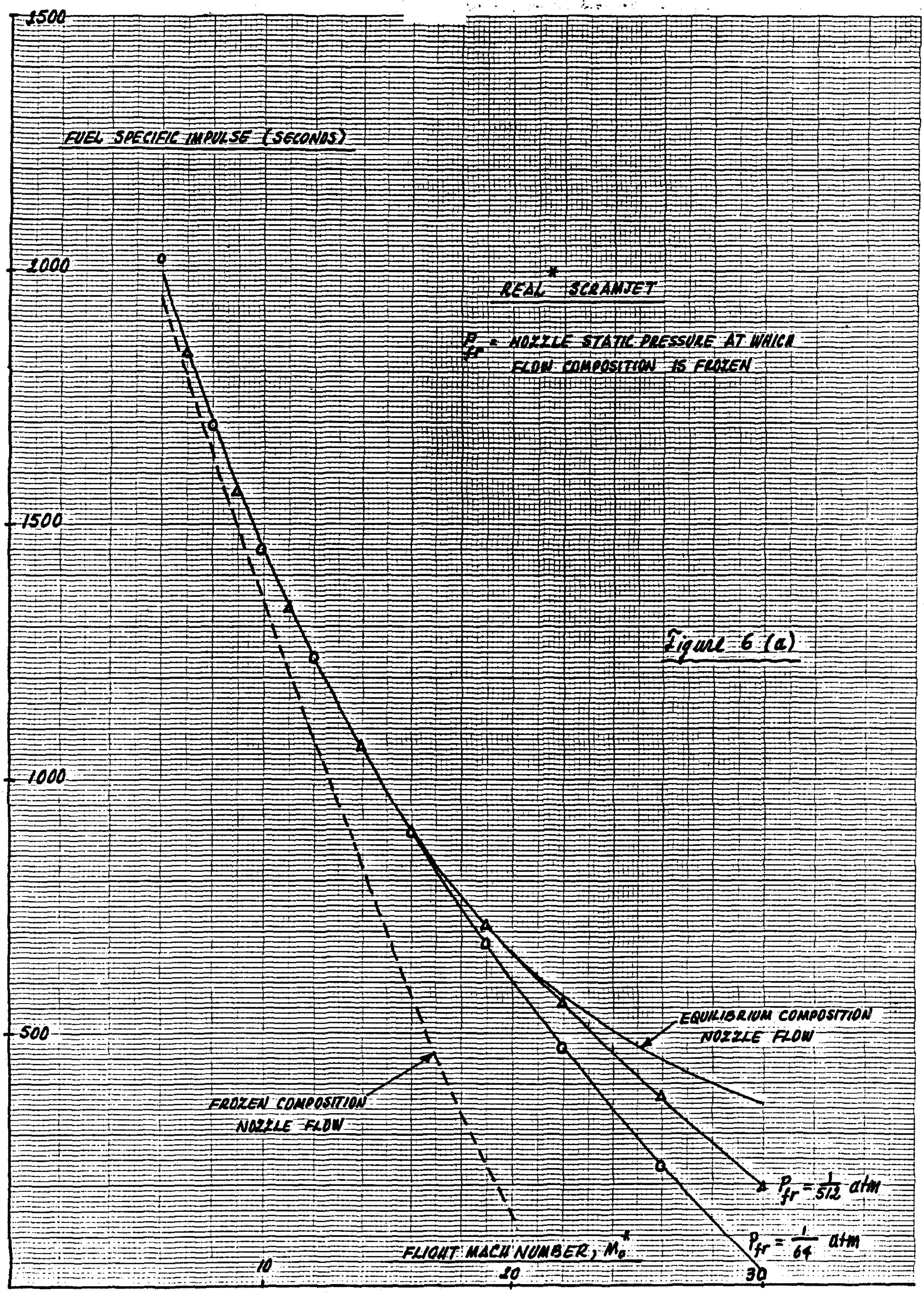
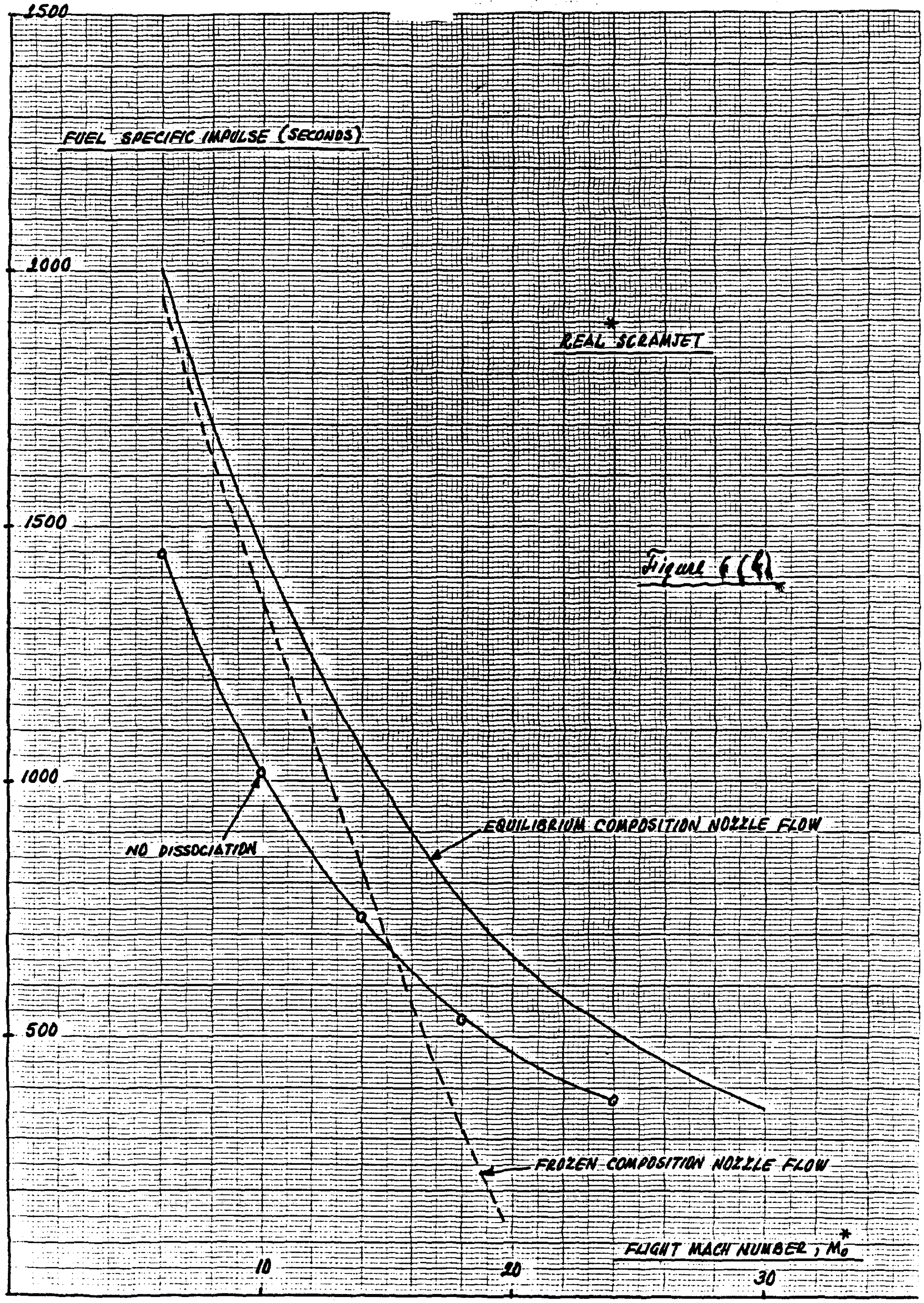

FUEL SPECIFIC IMPULSE (SECONDS)
IDEAL SCRAMJET
3000
2000
1000
EQUILIBRIUM COMPOSITION NOZZLE FLOW
NO DISSOCIATION OR FLOW IS FROZEN AT 1/64 atm or AT 1/512 atm
FROZEN COMPOSITION NOZZLE FLOW
FLIGHT MACH NUMBER, M_o^*
10
20
30

Figure 5


2500
FUEL SPECIFIC IMPULSE (SECONDS)
2000
REAL* SCRAMJET
P_{fr} = NOZZLE STATIC PRESSURE AT WHICH FLOW COMPOSITION IS FROZEN
1500
Figure 6 (a)
1000
500
EQUILIBRIUM COMPOSITION NOZZLE FLOW
FROZEN COMPOSITION NOZZLE FLOW
$P_{fr} = \frac{1}{512}$ atm
$P_{fr} = \frac{1}{64}$ atm
FLIGHT MACH NUMBER, M_0^*
10
20
30


FUEL SPECIFIC IMPULSE (SECONDS)
2500
2000
1500
1000
500
REAL SCRAMJET*
Figure 6 (b)
NO DISSOCIATION
EQUILIBRIUM COMPOSITION NOZZLE FLOW
FROZEN COMPOSITION NOZZLE FLOW
FLIGHT MACH NUMBER, M_0*
10
20
30

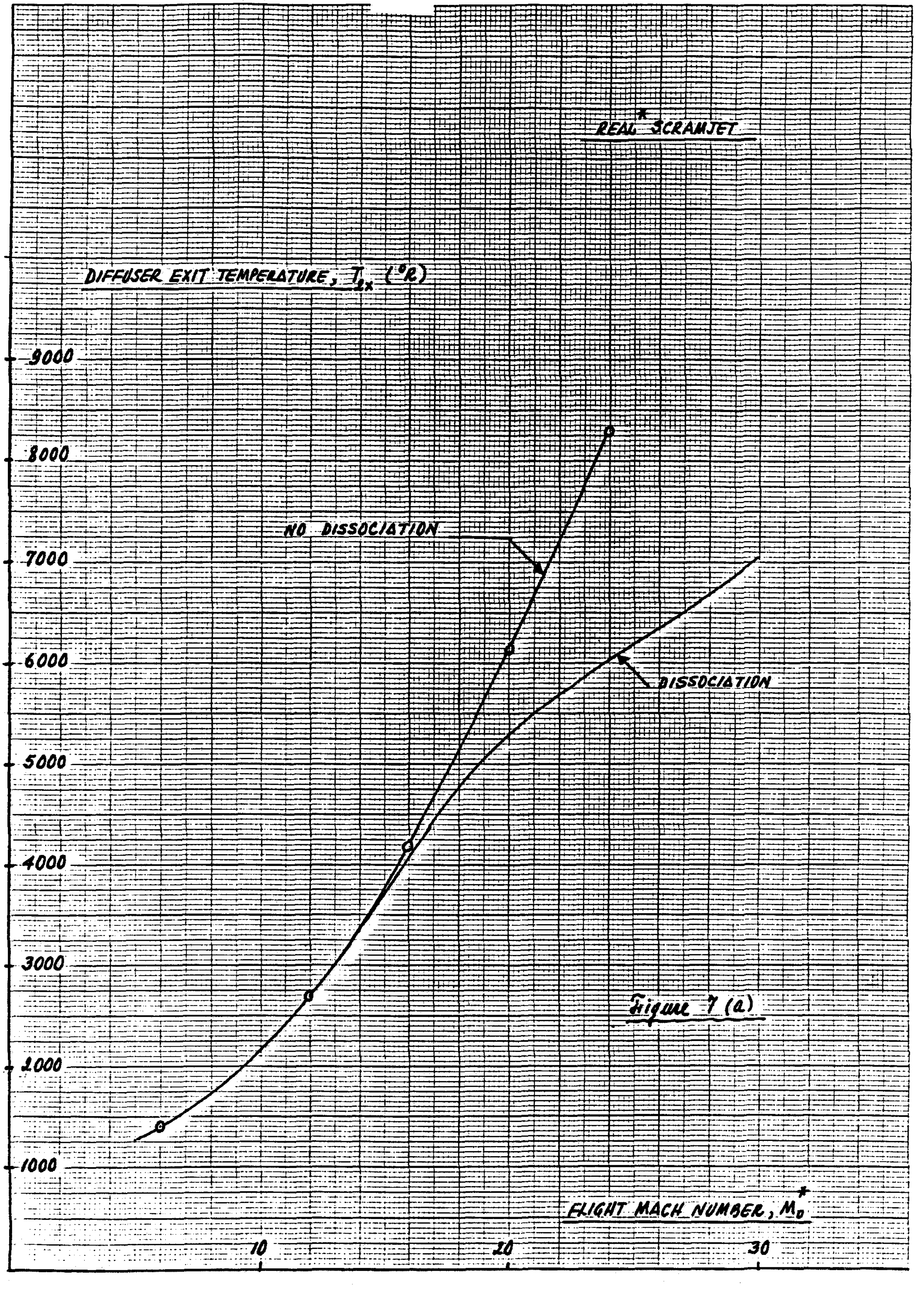

REAL* SCRAMJET
DIFFUSER EXIT TEMPERATURE, T_{2x} (°R)
9000
8000
7000
6000
5000
4000
3000
2000
1000
NO DISSOCIATION
DISSOCIATION
FLIGHT MACH NUMBER, M_0^*
10
20
30

Figure 7 (a)

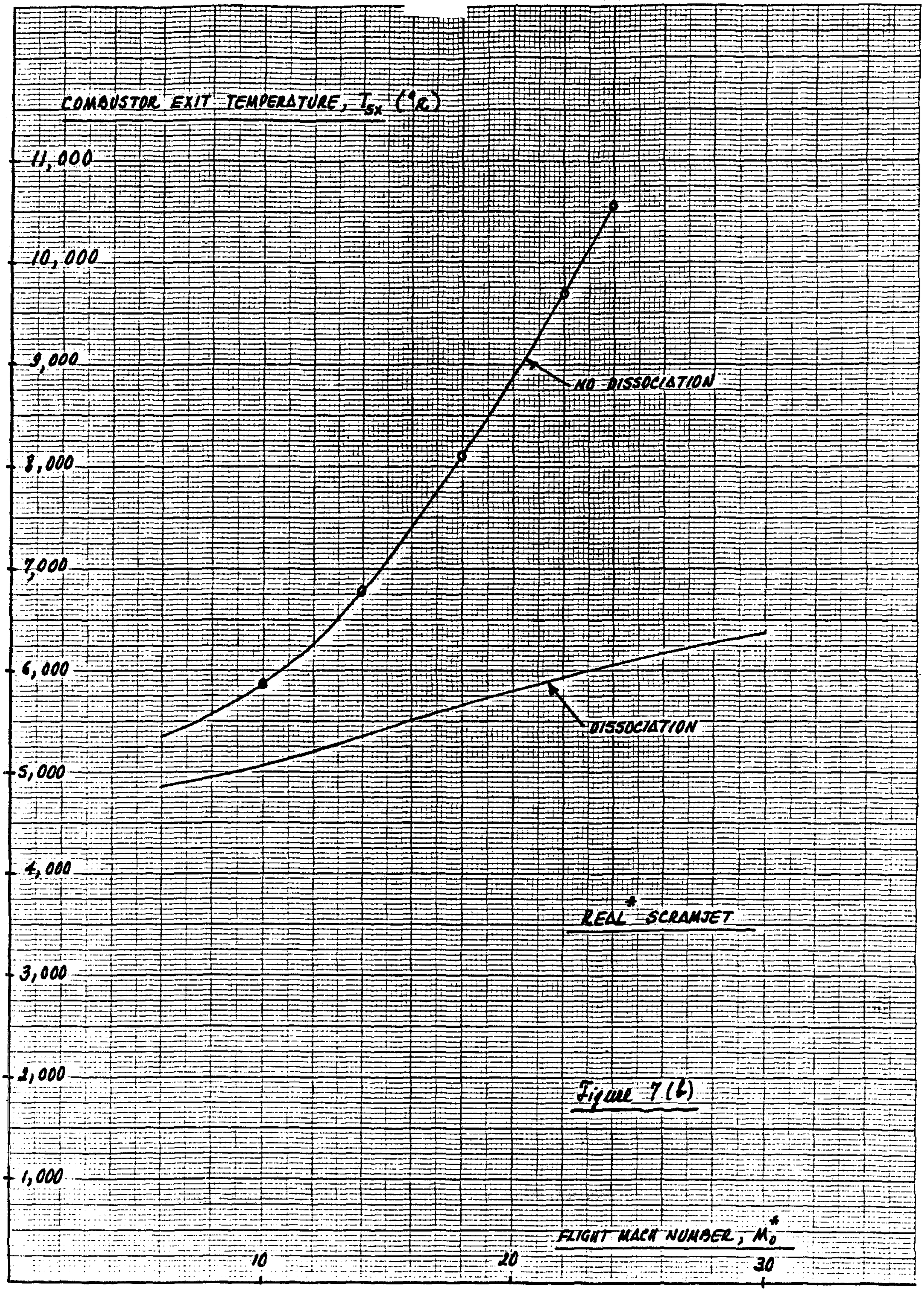

COMBUSTOR EXIT TEMPERATURE, T_{5x} (°R)
11,000
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
NO DISSOCIATION
DISSOCIATION
REAL* SCRAMJET
FLIGHT MACH NUMBER, M_0^*
10
20
30

Figure 7(b)

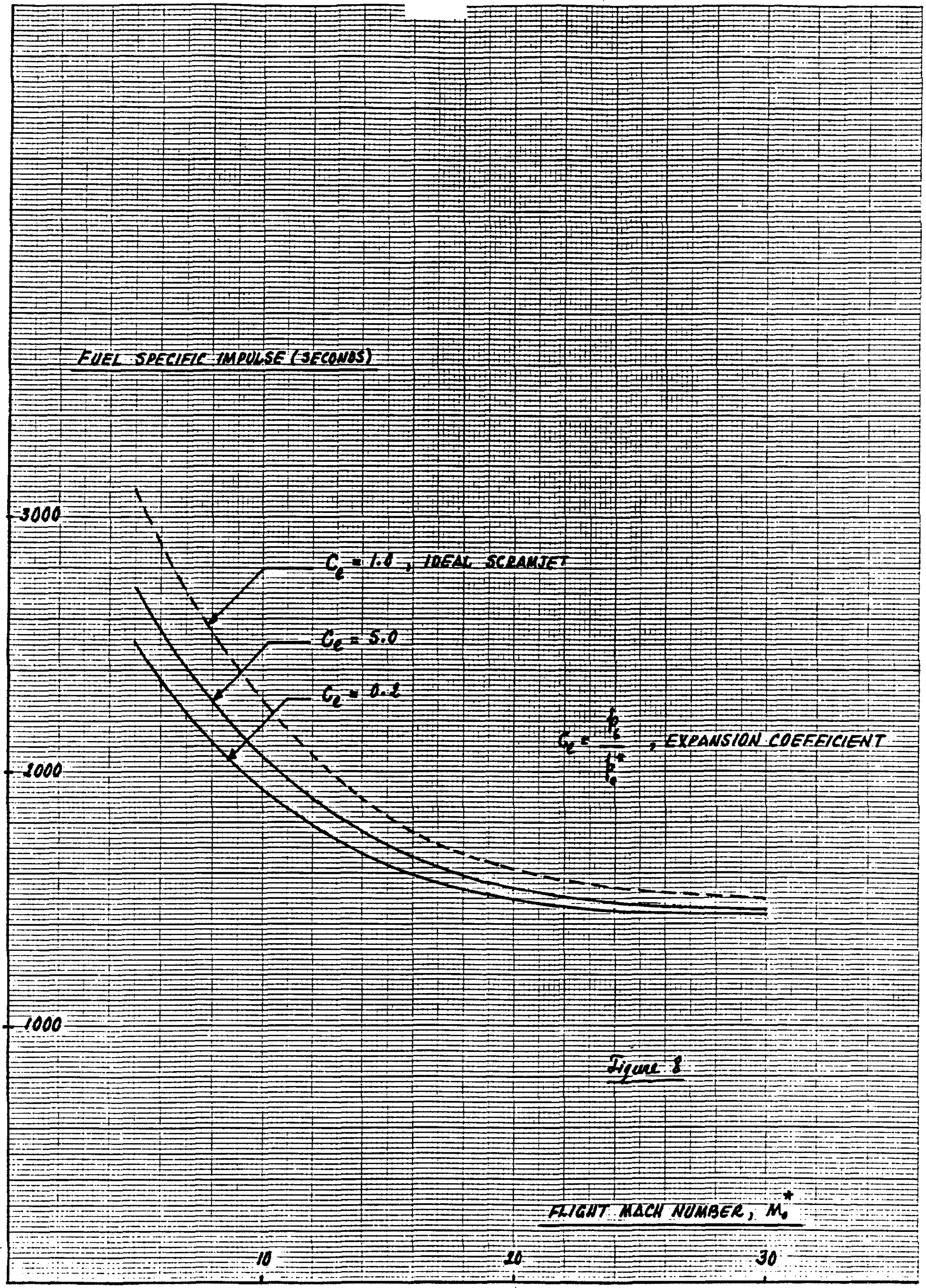

FUEL SPECIFIC IMPULSE (SECONDS)
3000
2000
1000
C_e = 1.0, IDEAL SCRAMJET
C_e = 5.0
C_e = 0.2
$C_e = \frac{p_e}{p_o^*}$, EXPANSION COEFFICIENT
FLIGHT MACH NUMBER, M_o^*
10
20
30

Figure 8

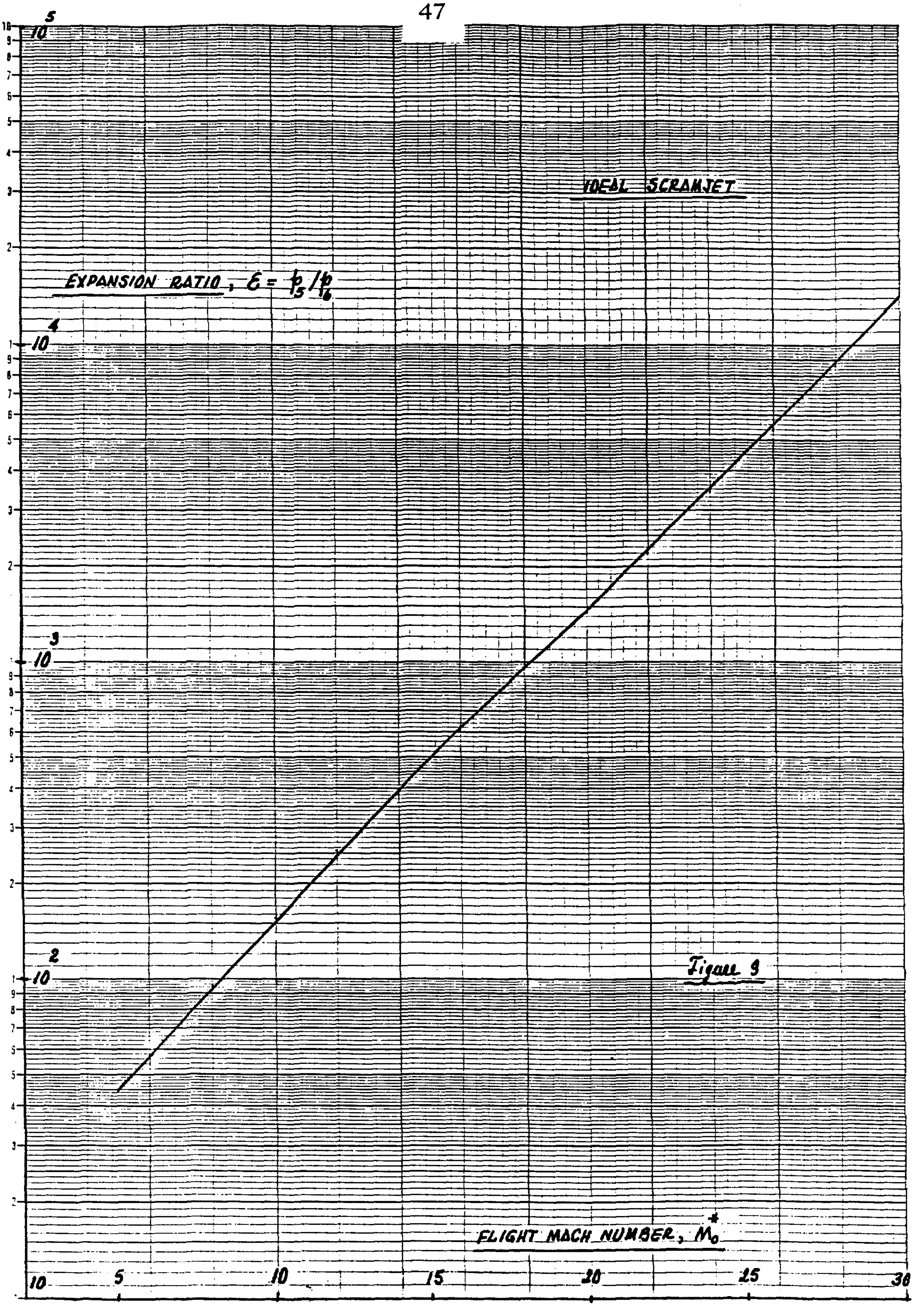

IDEAL SCRAMJET
EXPANSION RATIO, $\mathcal{E} = p_5/p_6$
10^5
10^4
10^3
10^2
10
5
10
15
20
25
30
FLIGHT MACH NUMBER, M_0^*

Figure 9

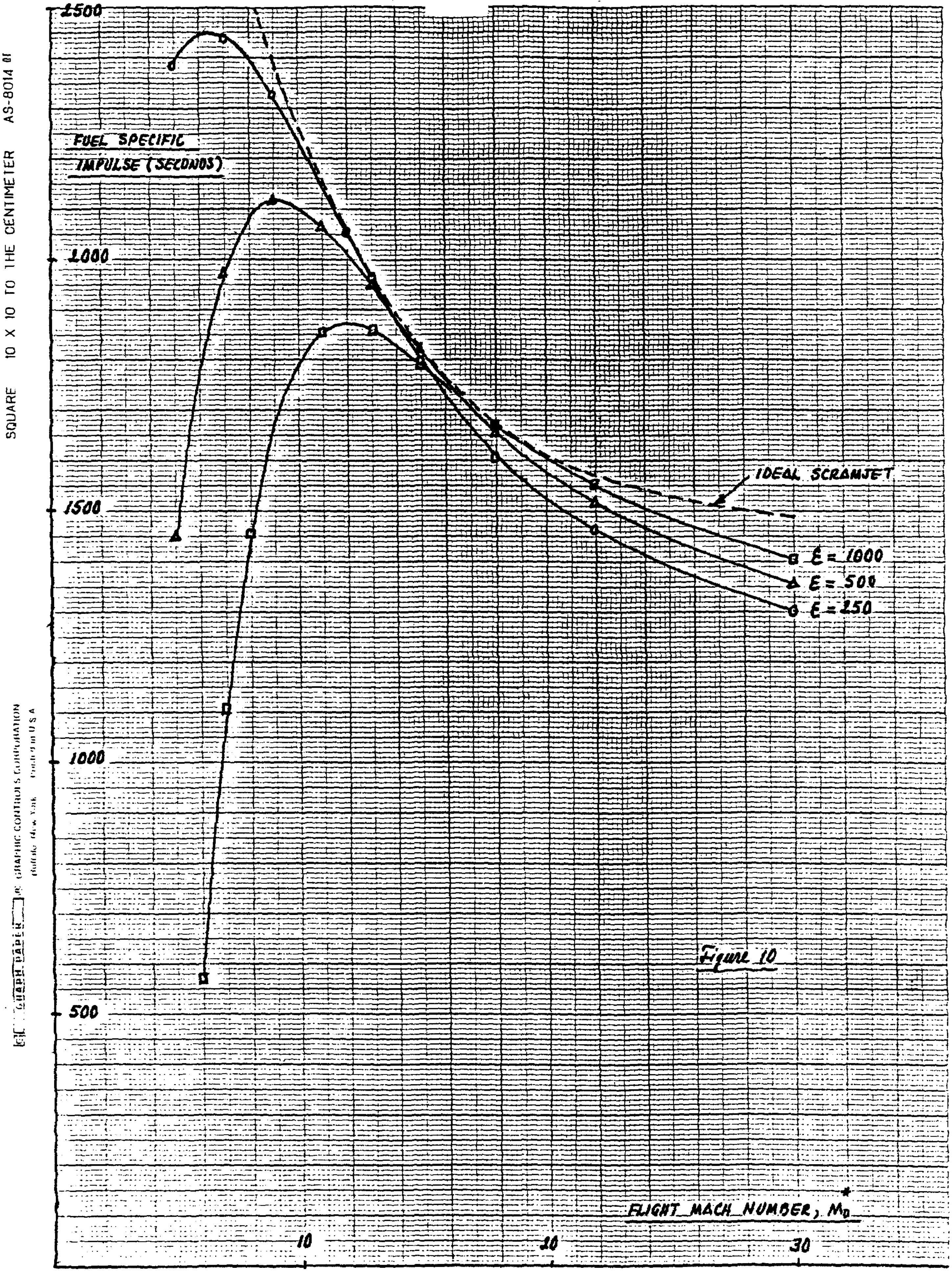

FUEL SPECIFIC IMPULSE (SECONDS)
2500
2000
1500
1000
500
IDEAL SCRAMJET
E = 1000
E = 500
E = 250
FLIGHT MACH NUMBER, M_0
10
20
30
SQUARE 10 X 10 TO THE CENTIMETER AS-8014 01
GRAPH PAPER GRAPHIC CONTROLS CORPORATION

Figure 10

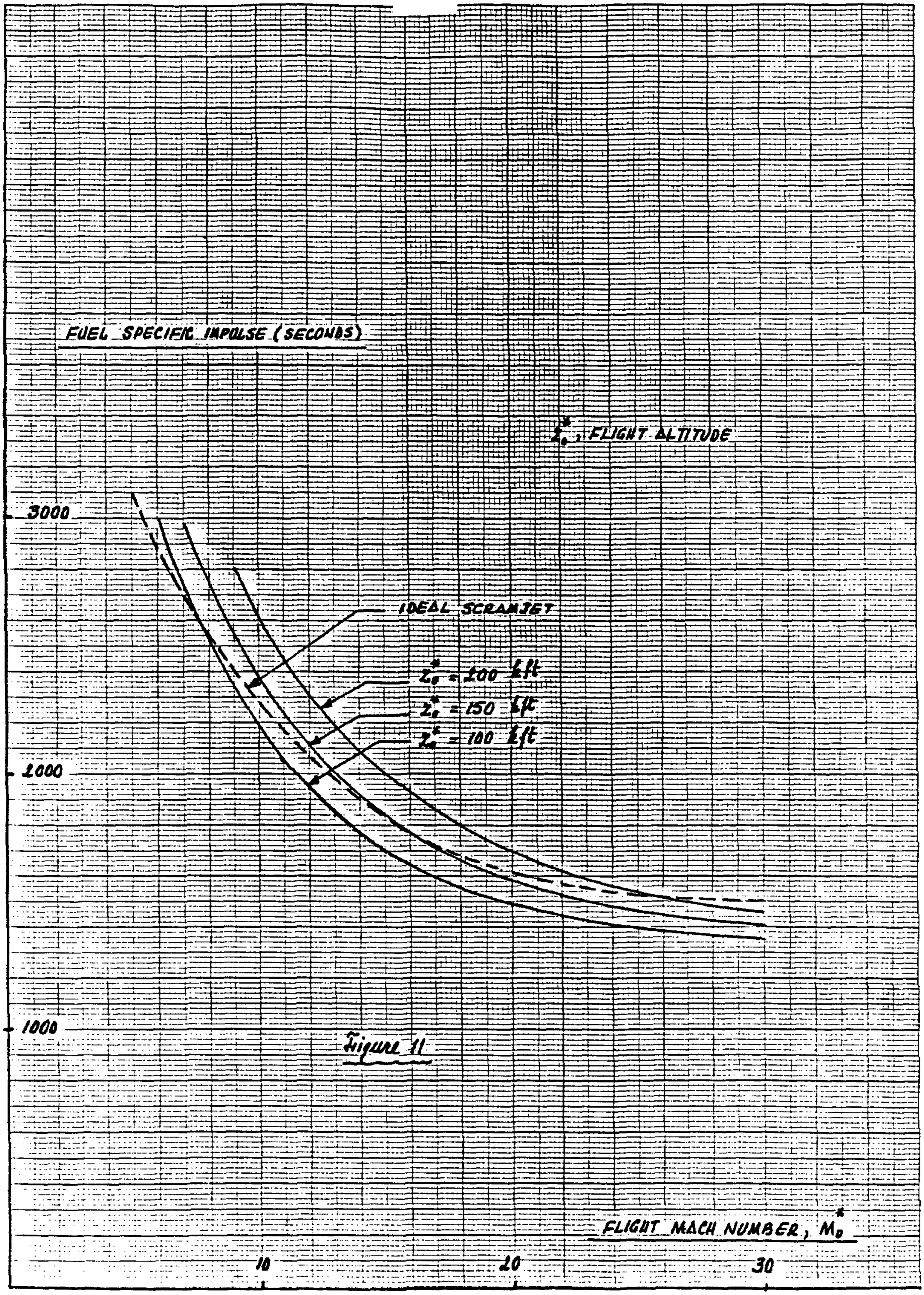

FUEL SPECIFIC IMPULSE (SECONDS)
z_0^*, FLIGHT ALTITUDE
3000
2000
1000
IDEAL SCRAMJET
$z_0^* = 200$ kft
$z_0^* = 150$ kft
$z_0^* = 100$ kft
10
20
30
FLIGHT MACH NUMBER, M_0^*

Figure 11

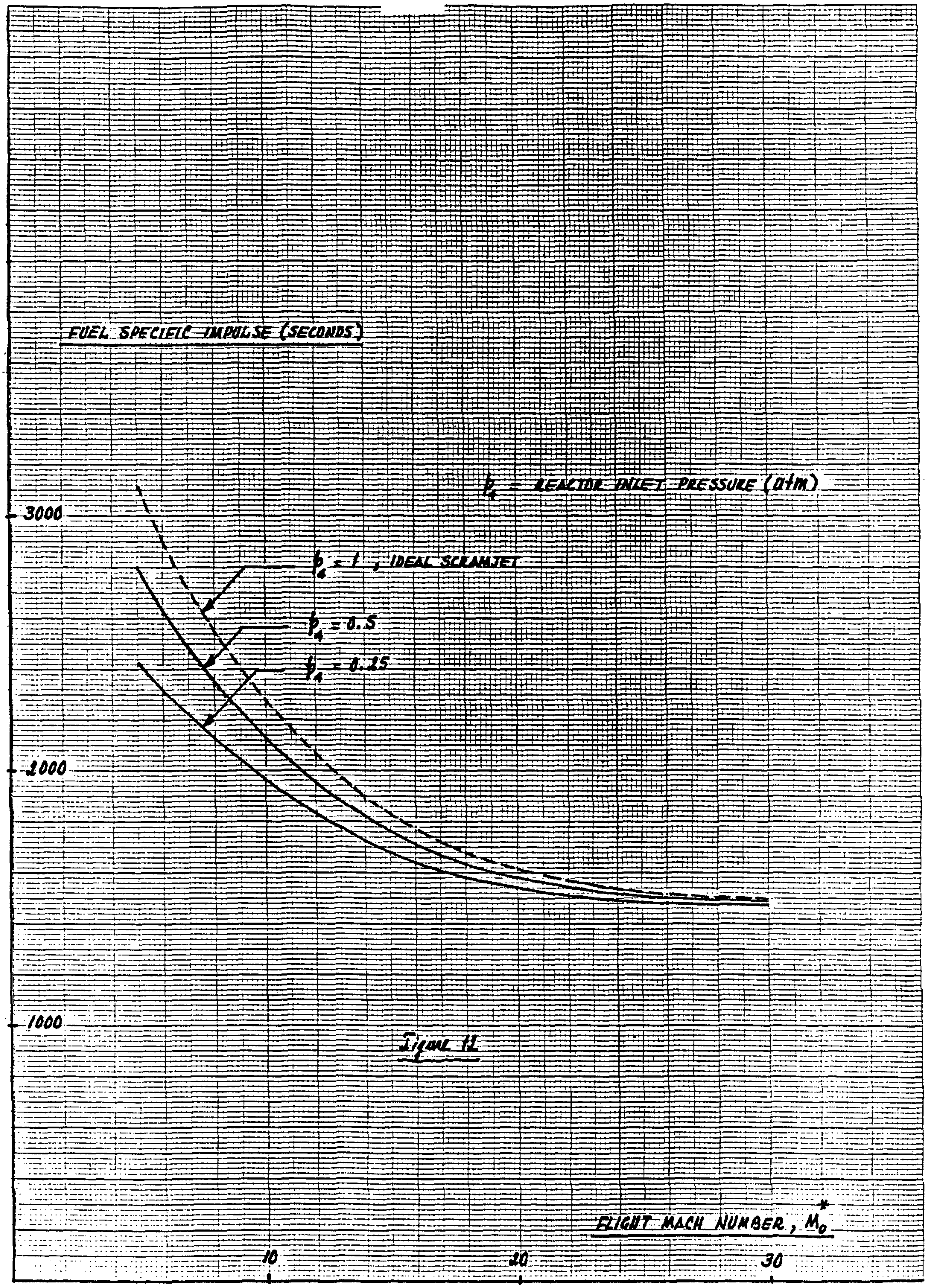

FUEL SPECIFIC IMPULSE (SECONDS)
p_4 = REACTOR INLET PRESSURE (atm)
3000
$p_4 = 1$, IDEAL SCRAMJET
$p_4 = 0.5$
$p_4 = 0.25$
2000
1000
FLIGHT MACH NUMBER, M_0^*
10
20
30

Figure 11

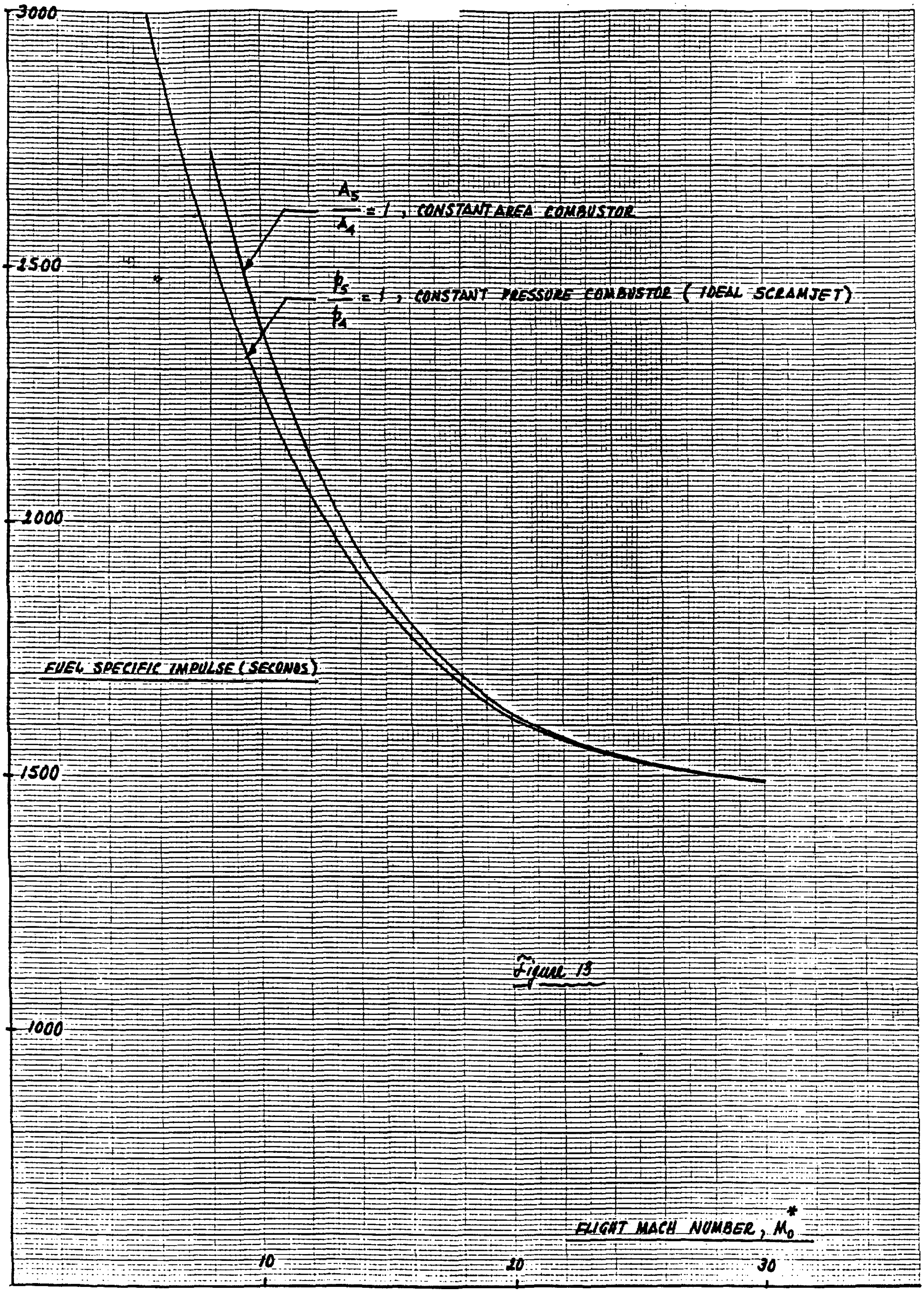

3000
2500
2000
1500
1000
$\frac{A_5}{A_4} = 1$, CONSTANT AREA COMBUSTOR
$\frac{p_5}{p_4} = 1$, CONSTANT PRESSURE COMBUSTOR (IDEAL SCRAMJET)
FUEL SPECIFIC IMPULSE (SECONDS)
FLIGHT MACH NUMBER, M_0^*
10
20
30

Figure 13

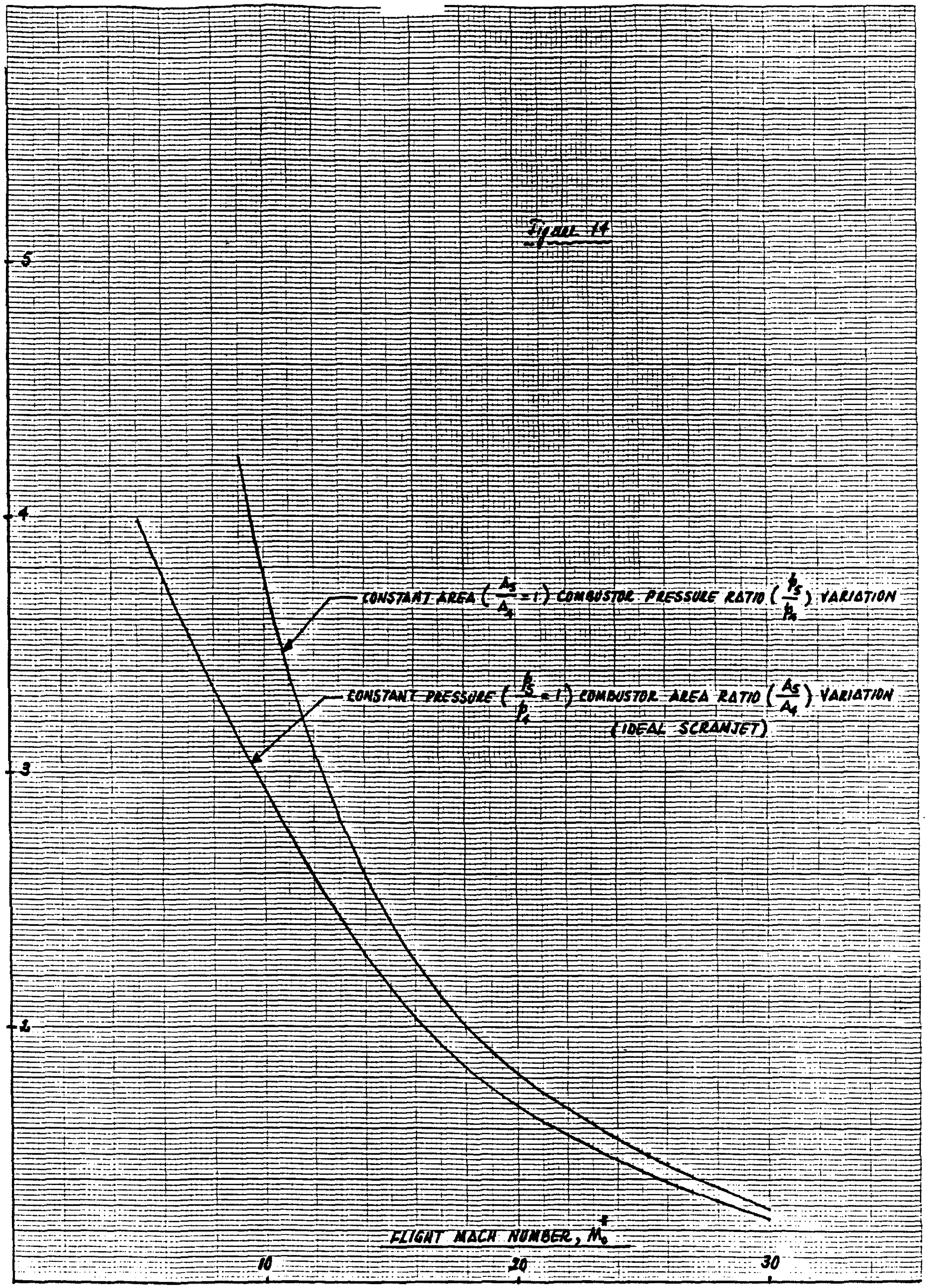
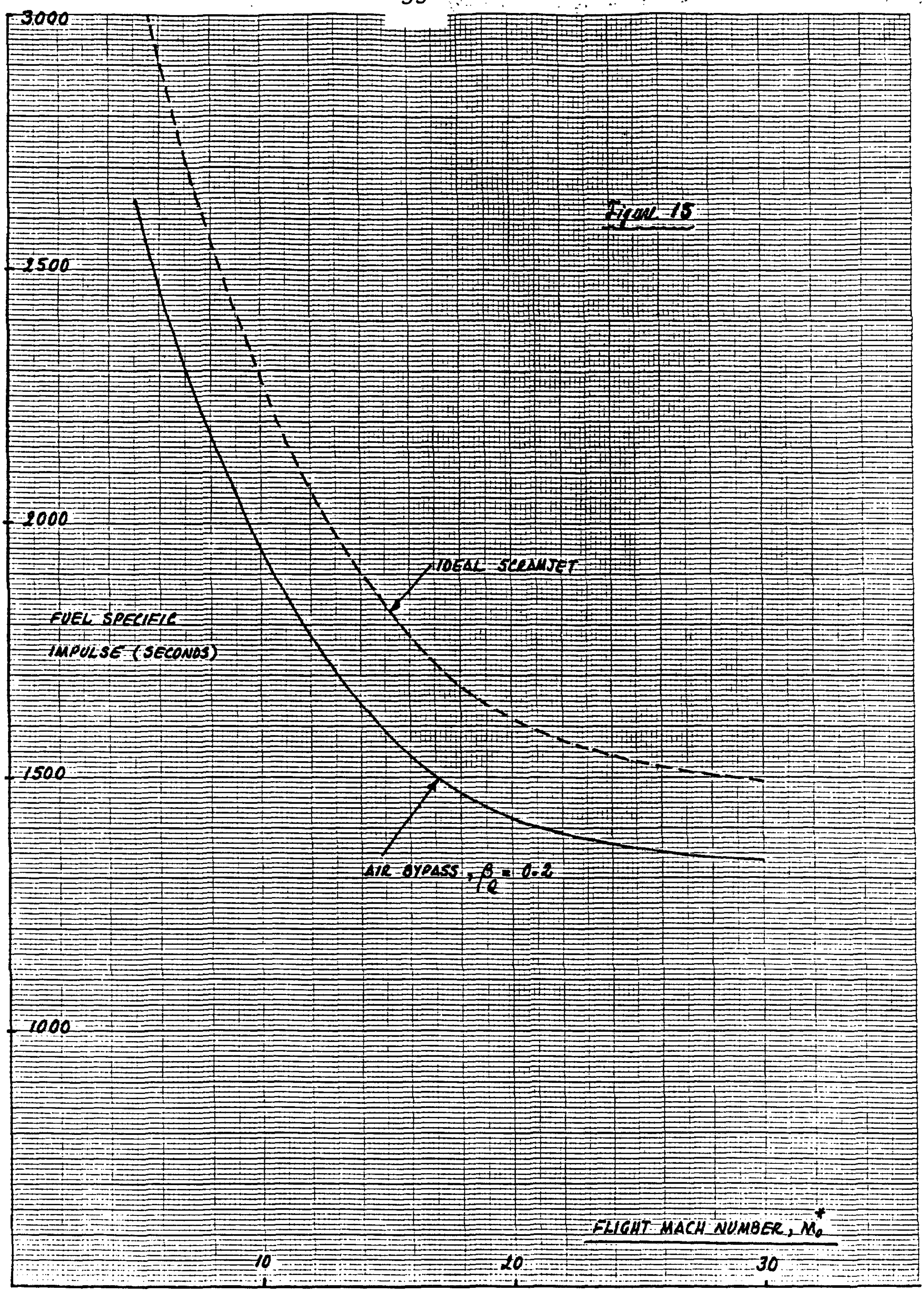


Figure 14
CONSTANT AREA $\left(\frac{A_5}{A_4}=1\right)$ COMBUSTOR PRESSURE RATIO $\left(\frac{p_5}{p_4}\right)$ VARIATION
CONSTANT PRESSURE $\left(\frac{p_5}{p_4}=1\right)$ COMBUSTOR AREA RATIO $\left(\frac{A_5}{A_4}\right)$ VARIATION
(IDEAL SCRAMJET)
FLIGHT MACH NUMBER, M_0
10
20
30
2
3
4
5



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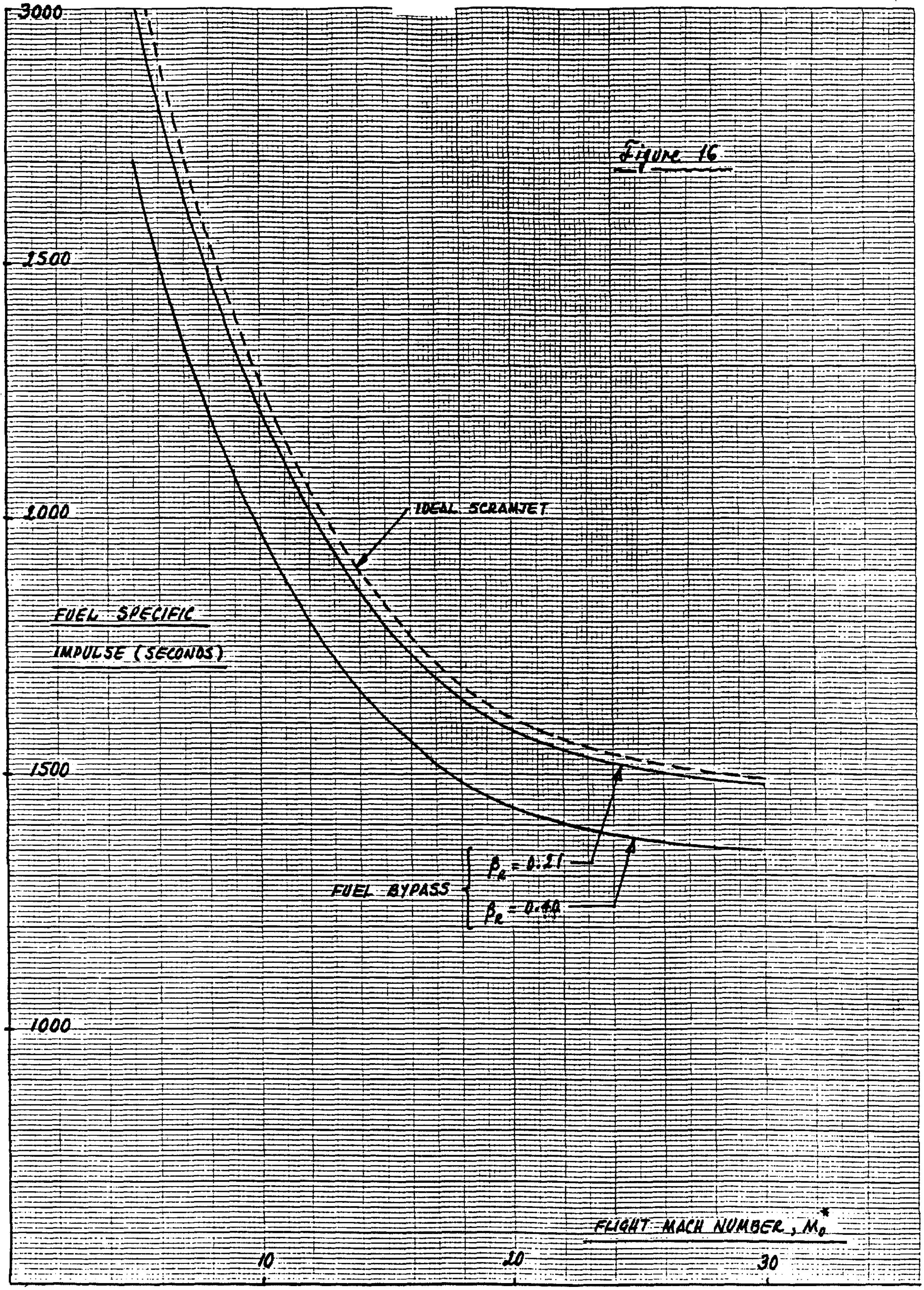
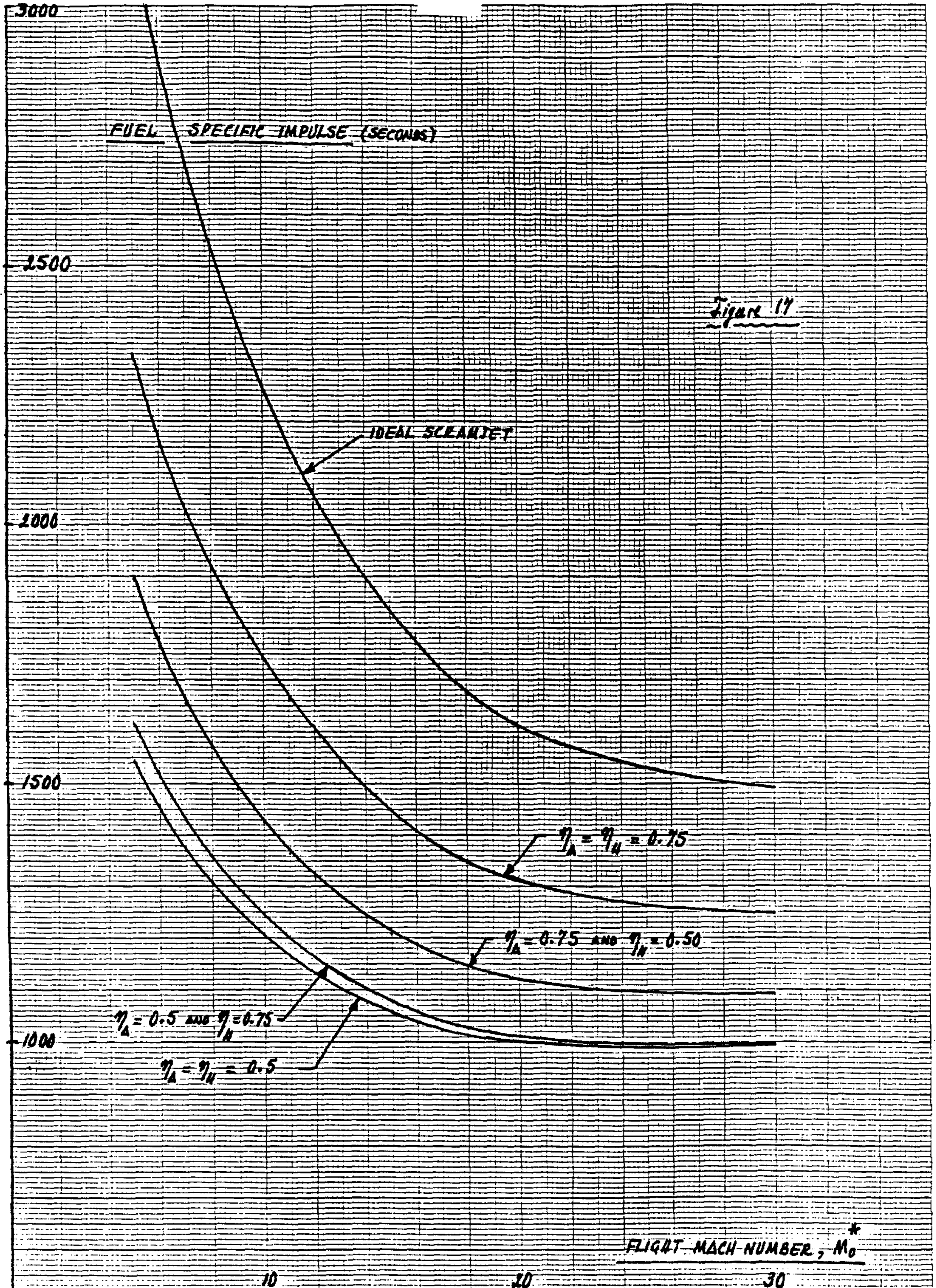


Figure 16
3000
2500
2000
1500
1000
FUEL SPECIFIC
IMPULSE (SECONDS)
IDEAL SCRAMJET
FUEL BYPASS
$\beta_R = 0.21$
$\beta_R = 0.40$
FLIGHT MACH NUMBER, M_0^*
10
20
30


FUEL SPECIFIC IMPULSE (SECONDS)
3000
2500
2000
1500
1000
Figure 17
IDEAL SCRAMJET
$\eta_A = \eta_H = 0.75$
$\eta_A = 0.75$ AND $\eta_H = 0.50$
$\eta_A = 0.5$ AND $\eta_H = 0.75$
$\eta_A = \eta_H = 0.5$
FLIGHT MACH NUMBER, M_0^*
10
20
30

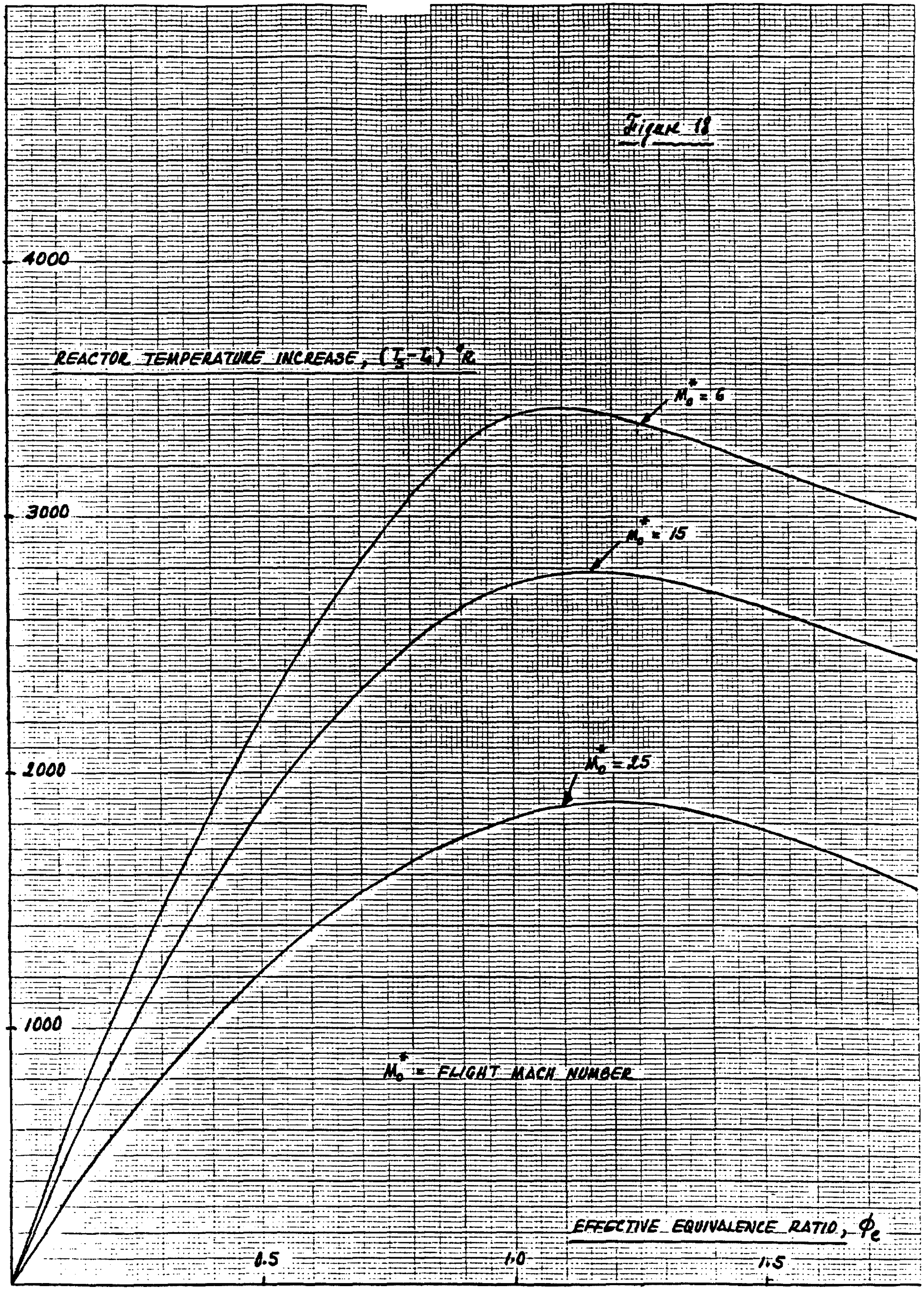
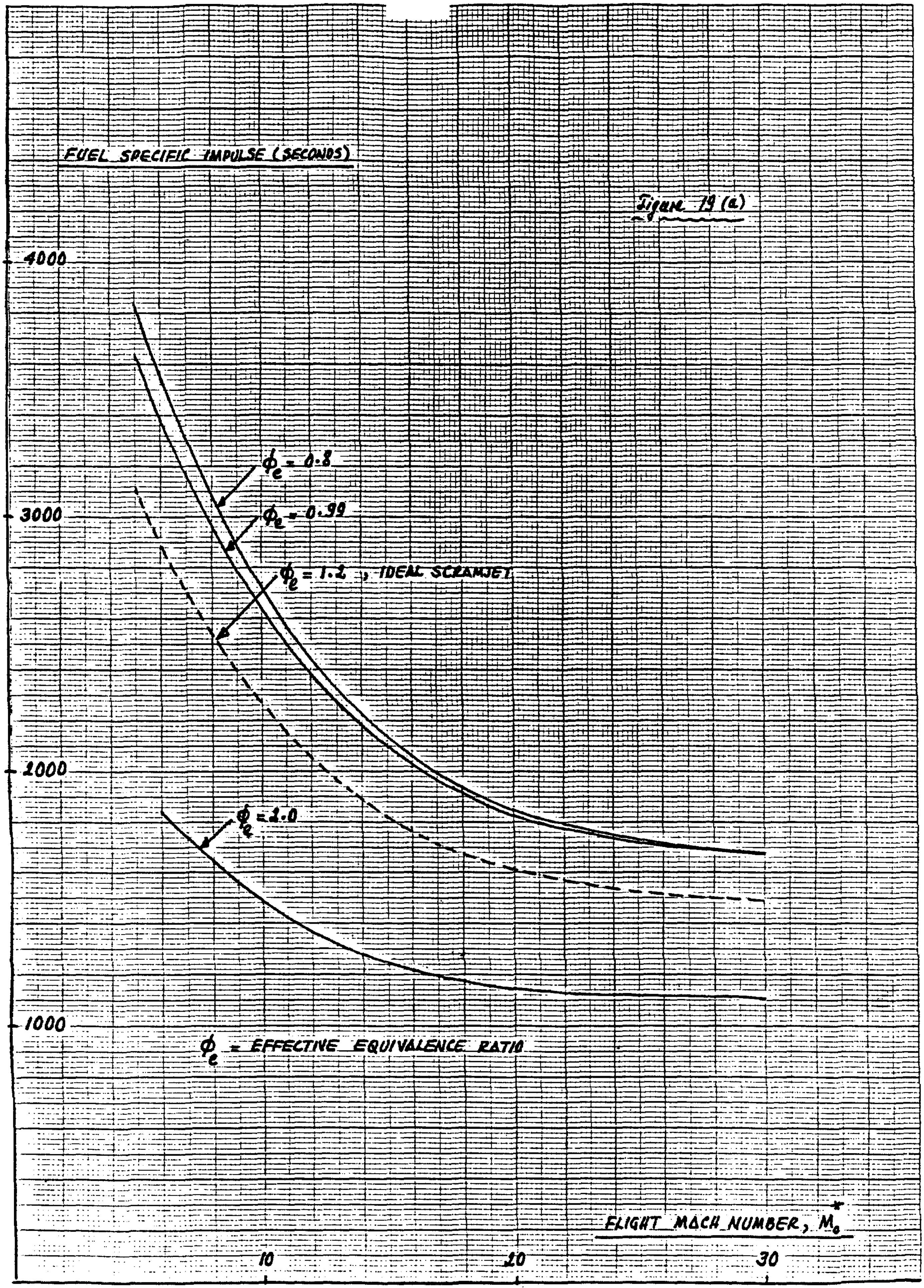
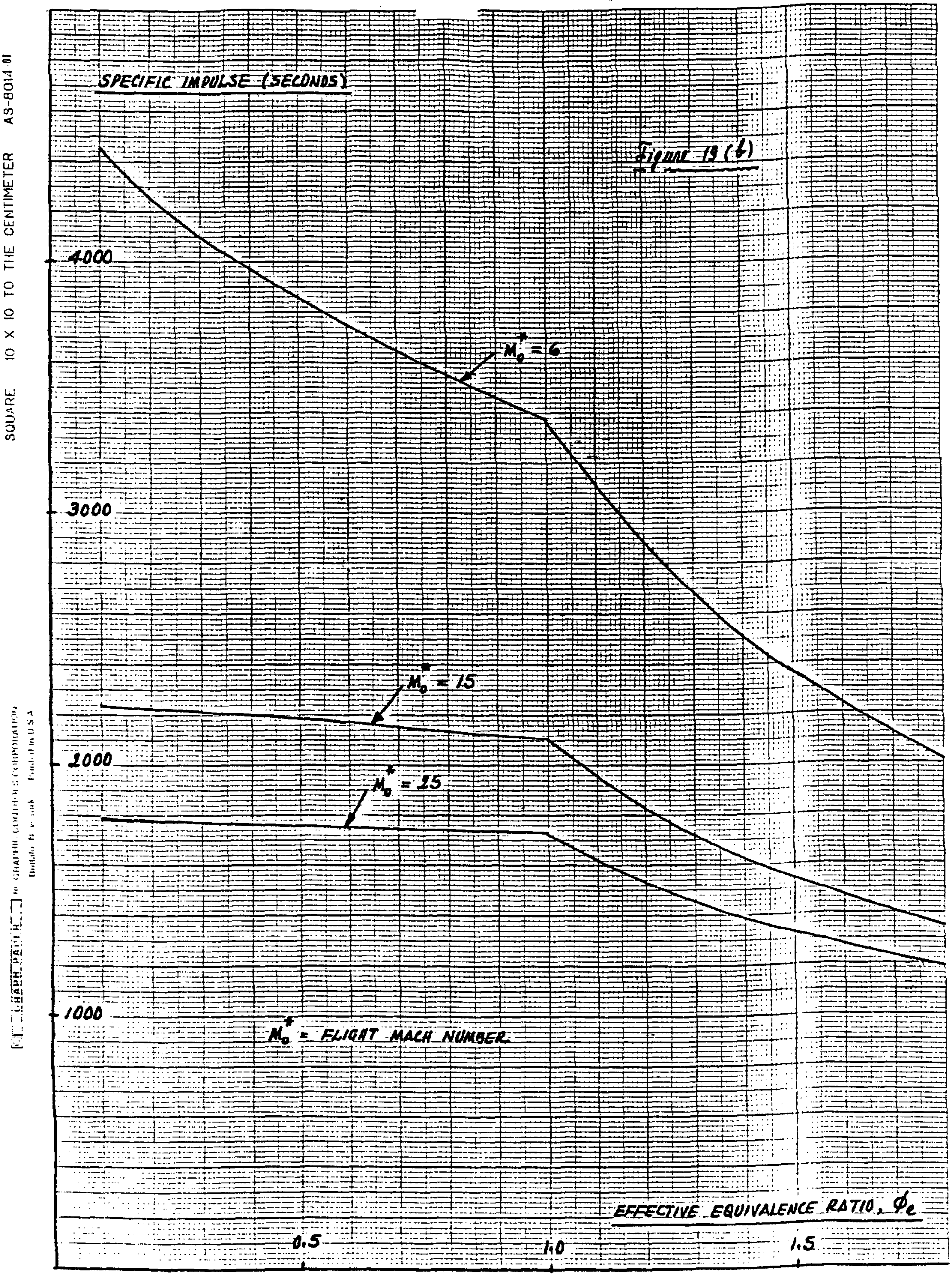


Figure 18
4000
3000
2000
1000
REACTOR TEMPERATURE INCREASE, $(T_5 - T_4)$ °R
$M_0^* = 6$
$M_0^* = 15$
$M_0^* = 25$
M_0^* = FLIGHT MACH NUMBER
EFFECTIVE EQUIVALENCE RATIO, ϕ_e
0.5
1.0
1.5


FUEL SPECIFIC IMPULSE (SECONDS)
Figure 19 (a)
4000
3000
2000
1000
ϕ_e = 0.8
ϕ_e = 0.99
ϕ_e = 1.2 , IDEAL SCRAMJET
ϕ_e = 2.0
ϕ_e = EFFECTIVE EQUIVALENCE RATIO
FLIGHT MACH NUMBER, M_0
10
20
30


SPECIFIC IMPULSE (SECONDS)
Figure 19 (b)
4000
3000
2000
1000
$M_0^* = 6$
$M_0^* = 15$
$M_0^* = 25$
M_0^* = FLIGHT MACH NUMBER
EFFECTIVE EQUIVALENCE RATIO, ϕ_e
0.5
1.0
1.5
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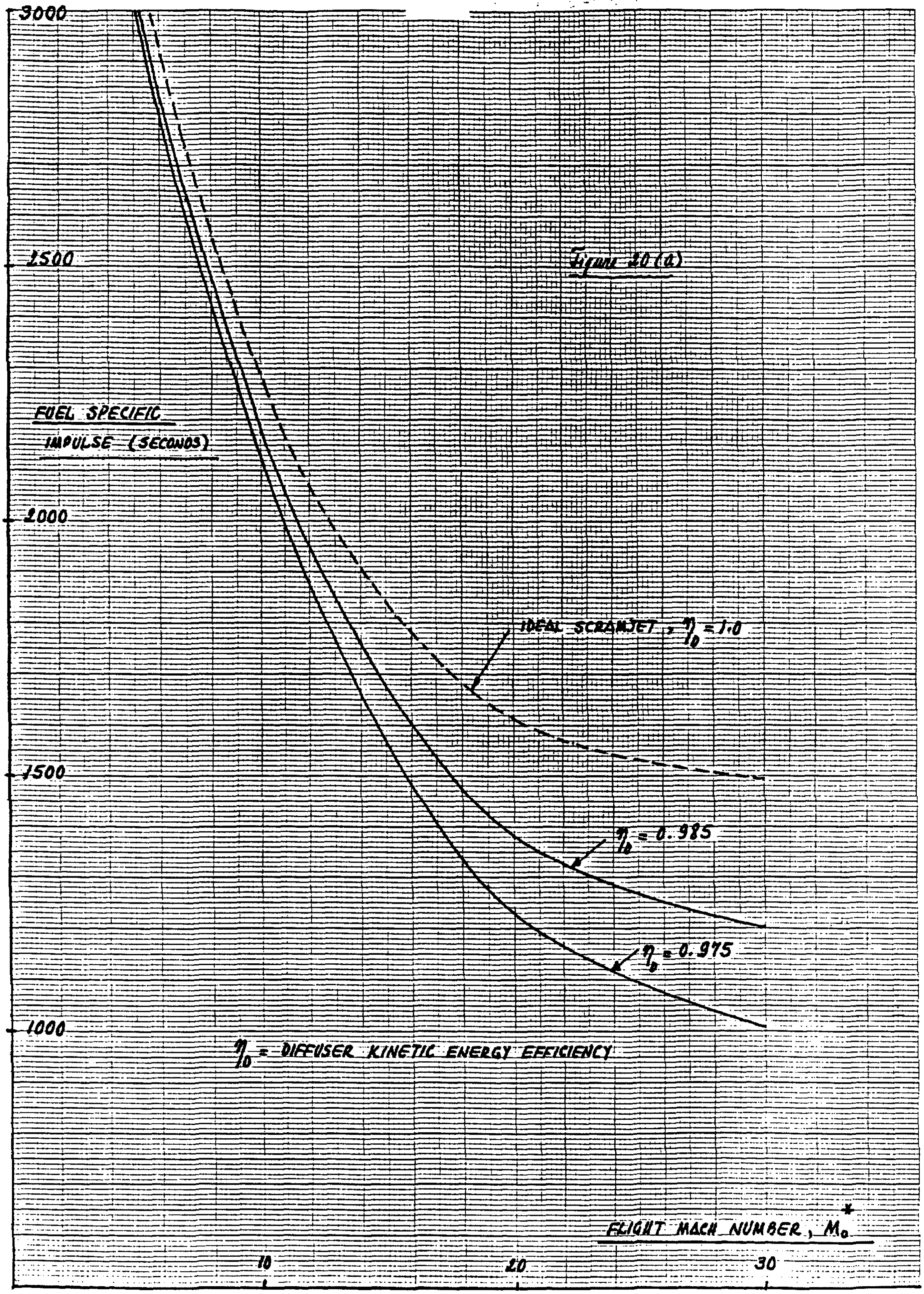

3000
2500
2000
1500
1000
Figure 20 (a)
FUEL SPECIFIC IMPULSE (SECONDS)
IDEAL SCRAMJET, $\eta_D = 1.0$
$\eta_D = 0.985$
$\eta_D = 0.975$
η_D = DIFFUSER KINETIC ENERGY EFFICIENCY
FLIGHT MACH NUMBER, M_0
10
20
30

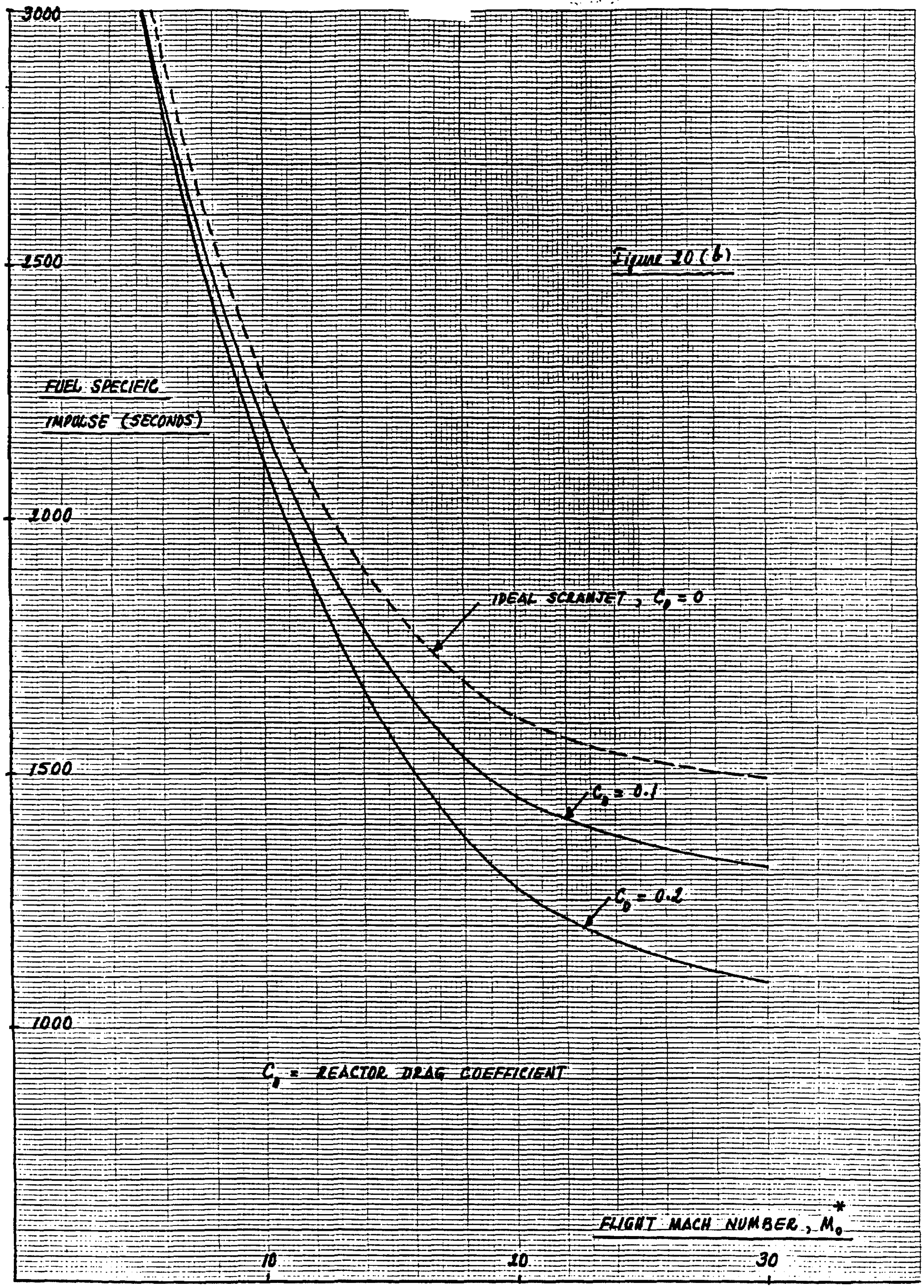
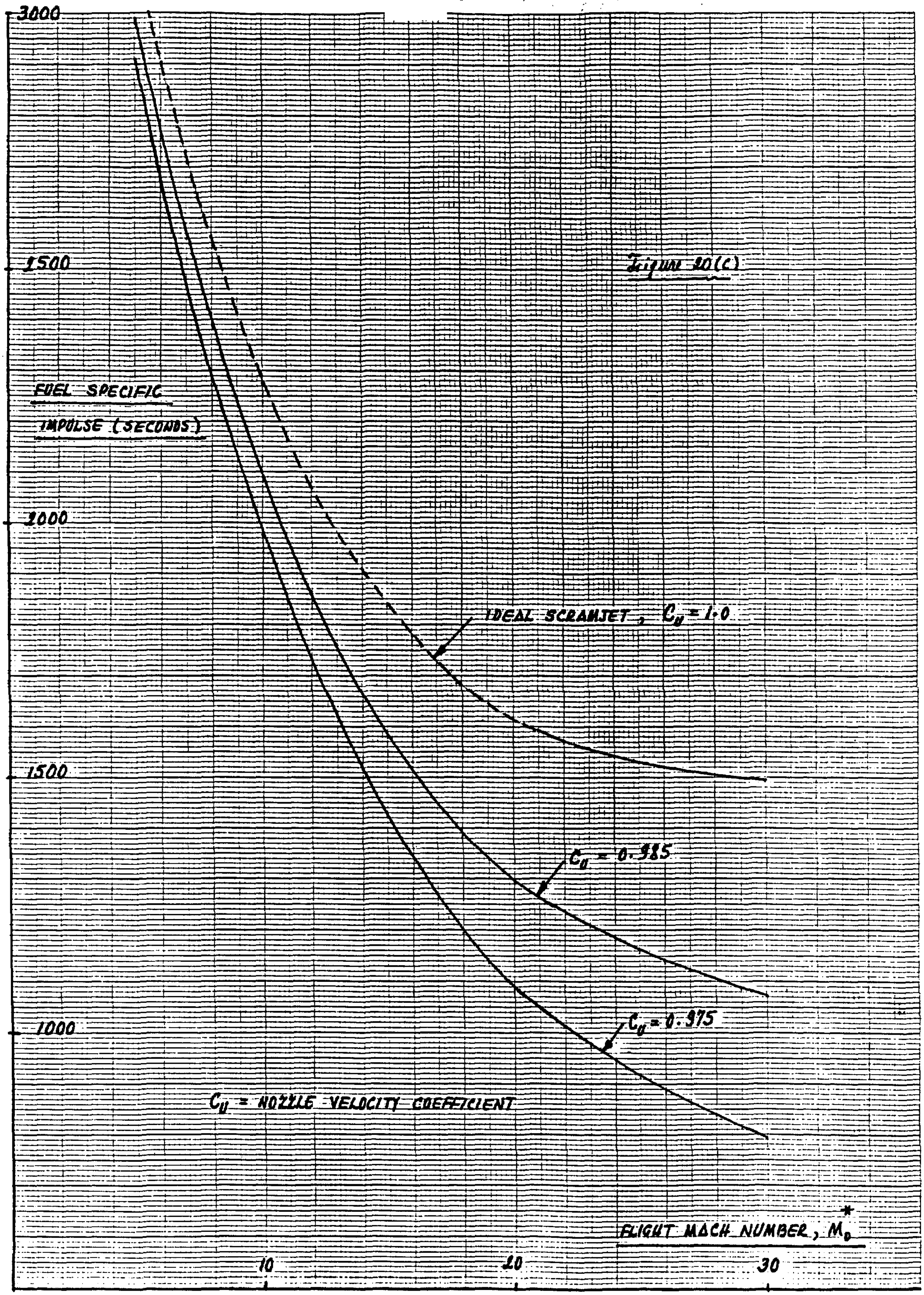
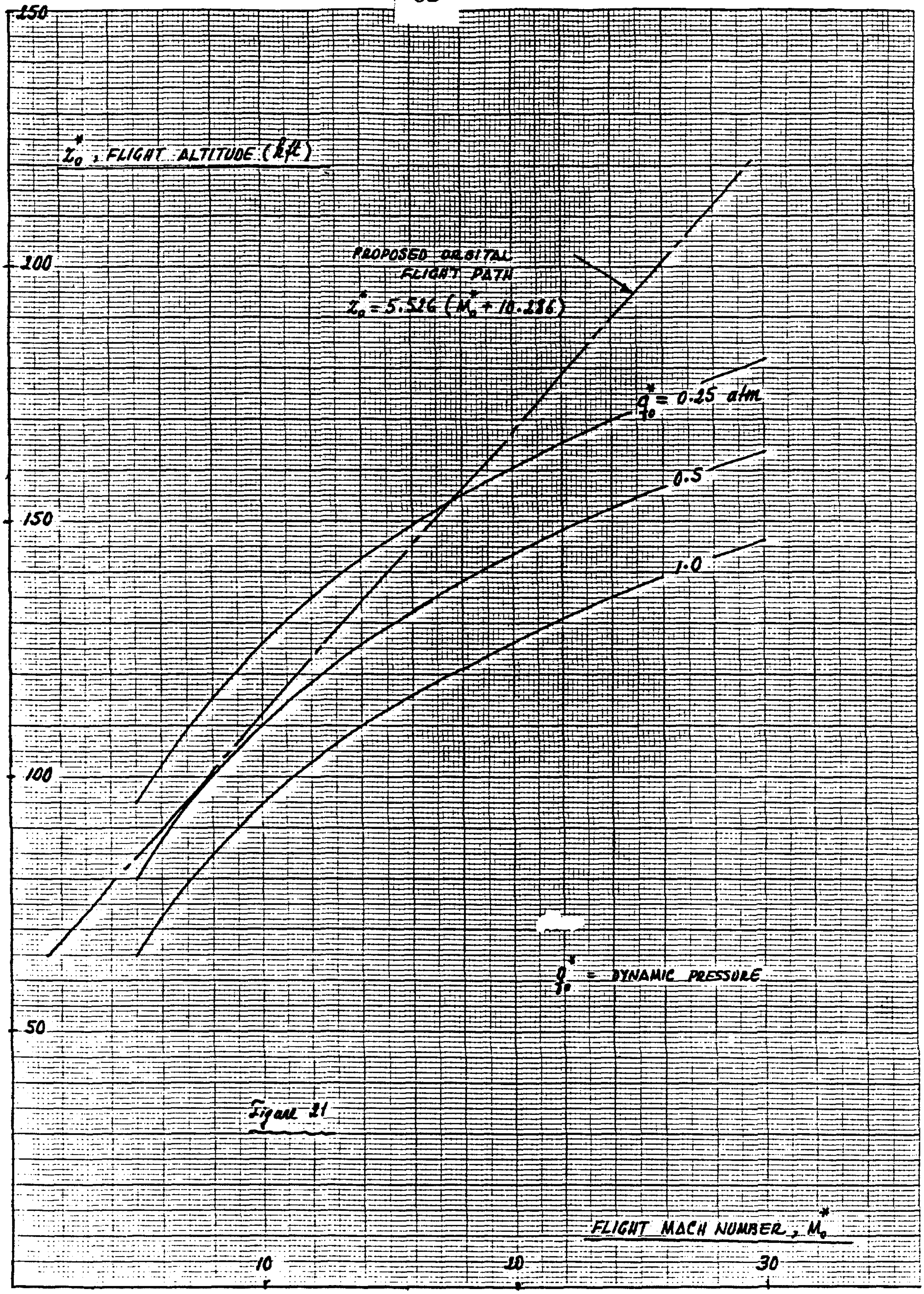


Figure 20 (b)
FUEL SPECIFIC IMPULSE (SECONDS)
3000
2500
2000
1500
1000
IDEAL SCRAMJET, $C_D = 0$
$C_D = 0.1$
$C_D = 0.2$
C_D = REACTOR DRAG COEFFICIENT
FLIGHT MACH NUMBER, M_0
10
20
30


3000
2500
2000
1500
1000
Figure 20(c)
FUEL SPECIFIC IMPULSE (SECONDS)
IDEAL SCRAMJET, $C_v = 1.0$
$C_v = 0.985$
$C_v = 0.975$
C_v = NOZZLE VELOCITY COEFFICIENT
FLIGHT MACH NUMBER, M_0^*
10
20
30



Z_0^*, FLIGHT ALTITUDE (kft)
250
200
150
100
50
PROPOSED ORBITAL FLIGHT PATH
$Z_0^* = 5.526(M_0^* + 10.286)$
$q_0^* = 0.25$ atm
0.5
1.0
q_0^* = DYNAMIC PRESSURE
FLIGHT MACH NUMBER, M_0^*
10
20
30

Figure 21

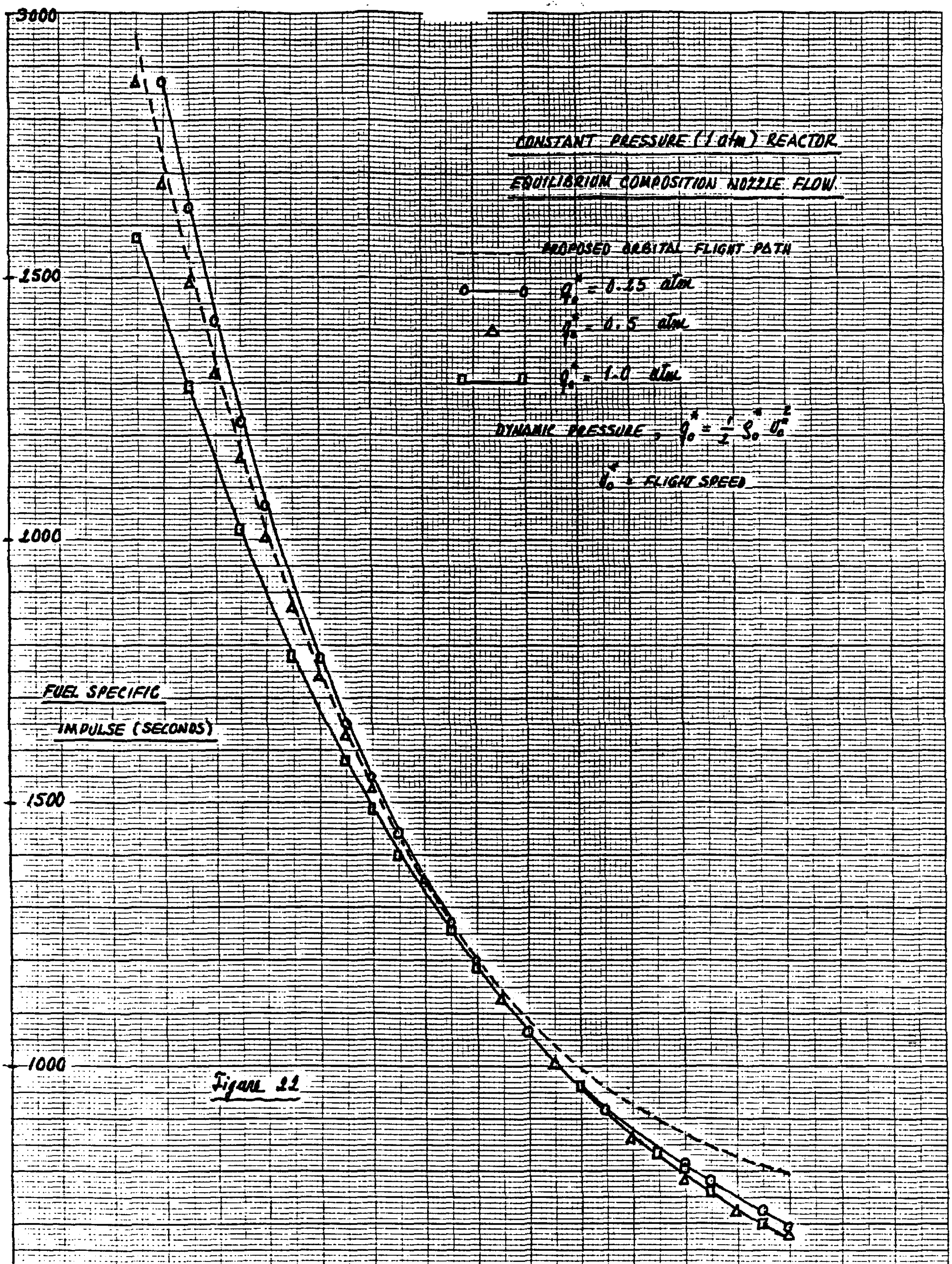
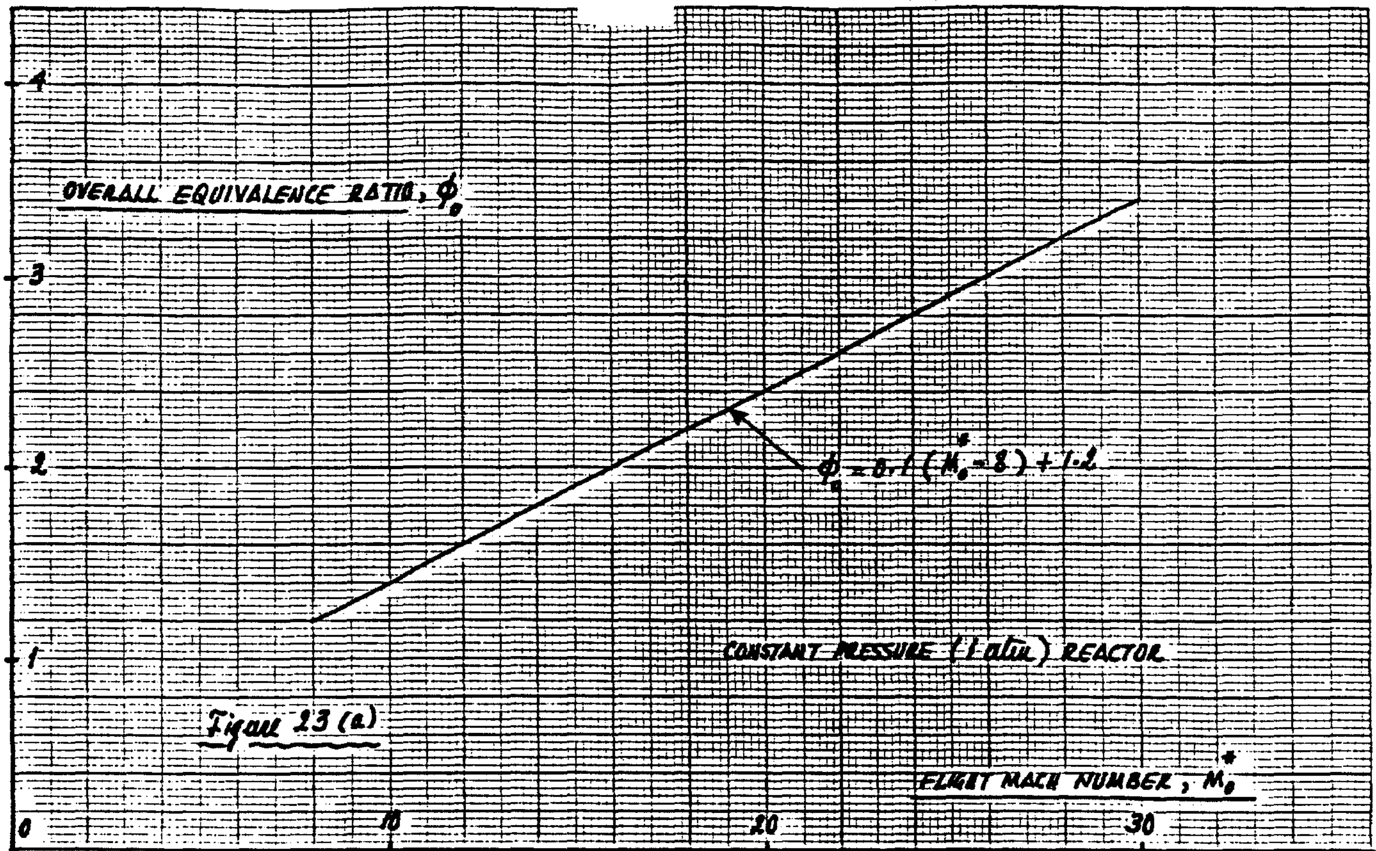
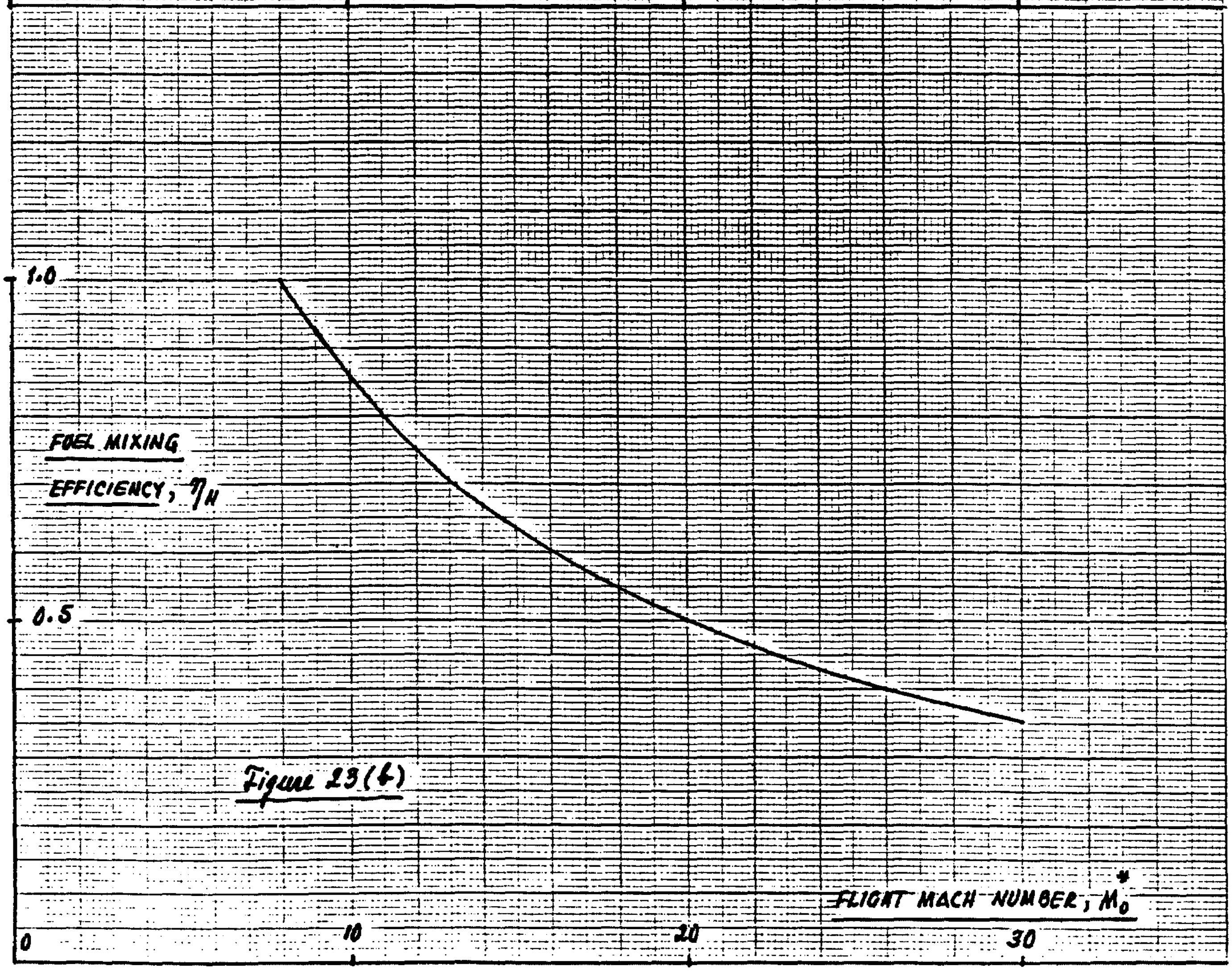

CONSTANT PRESSURE (1 atm) REACTOR
EQUILIBRIUM COMPOSITION NOZZLE FLOW
PROPOSED ORBITAL FLIGHT PATH
$q_0^* = 0.25$ atm
$q_0^* = 0.5$ atm
$q_0^* = 1.0$ atm
DYNAMIC PRESSURE, $q_0^* = \frac{1}{2} \rho_0^* U_0^{*2}$
U_0^* = FLIGHT SPEED
FUEL SPECIFIC IMPULSE (SECONDS)
FLIGHT MACH NUMBER, M_0^*
3000
2500
2000
1500
1000
500
5
10
20
30

Figure 21


OVERALL EQUIVALENCE RATIO, ϕ_o
4
3
2
1
$\phi_o = 0.1(M_o^* - 8) + 1.2$
CONSTANT PRESSURE (1 atm) REACTOR
Figure 23 (a)
FLIGHT MACH NUMBER, M_o^*
0
10
20
30


1.0
FUEL MIXING
EFFICIENCY, η_H
0.5
Figure 23 (b)
FLIGHT MACH NUMBER, M_o^*
0
10
20
30

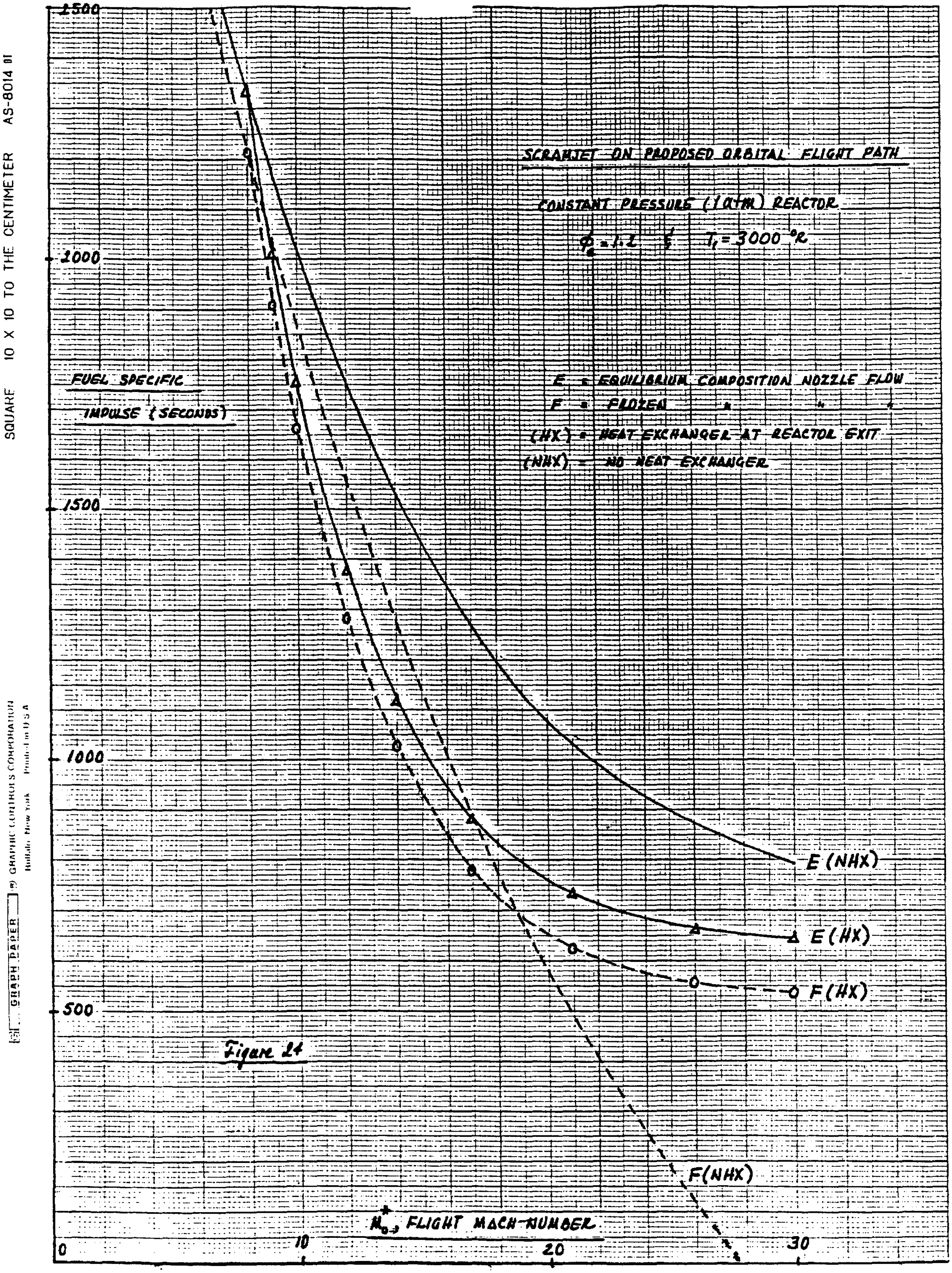

SCRAMJET ON PROPOSED ORBITAL FLIGHT PATH
CONSTANT PRESSURE (1 atm) REACTOR
$\phi = 1.2$ & $T_1 = 3000\,°R$
FUEL SPECIFIC IMPULSE (SECONDS)
E = EQUILIBRIUM COMPOSITION NOZZLE FLOW
F = FROZEN " " "
(HX) = HEAT EXCHANGER AT REACTOR EXIT
(NHX) = NO HEAT EXCHANGER
2500
2000
1500
1000
500
0
10
20
30
E (NHX)
E (HX)
F (HX)
F(NHX)
M_{0}, FLIGHT MACH NUMBER
SQUARE 10 X 10 TO THE CENTIMETER AS-8014 01
GRAPH PAPER GRAPHIC CONTROLS CORPORATION Buffalo, New York Printed in U.S.A.

Figure 24

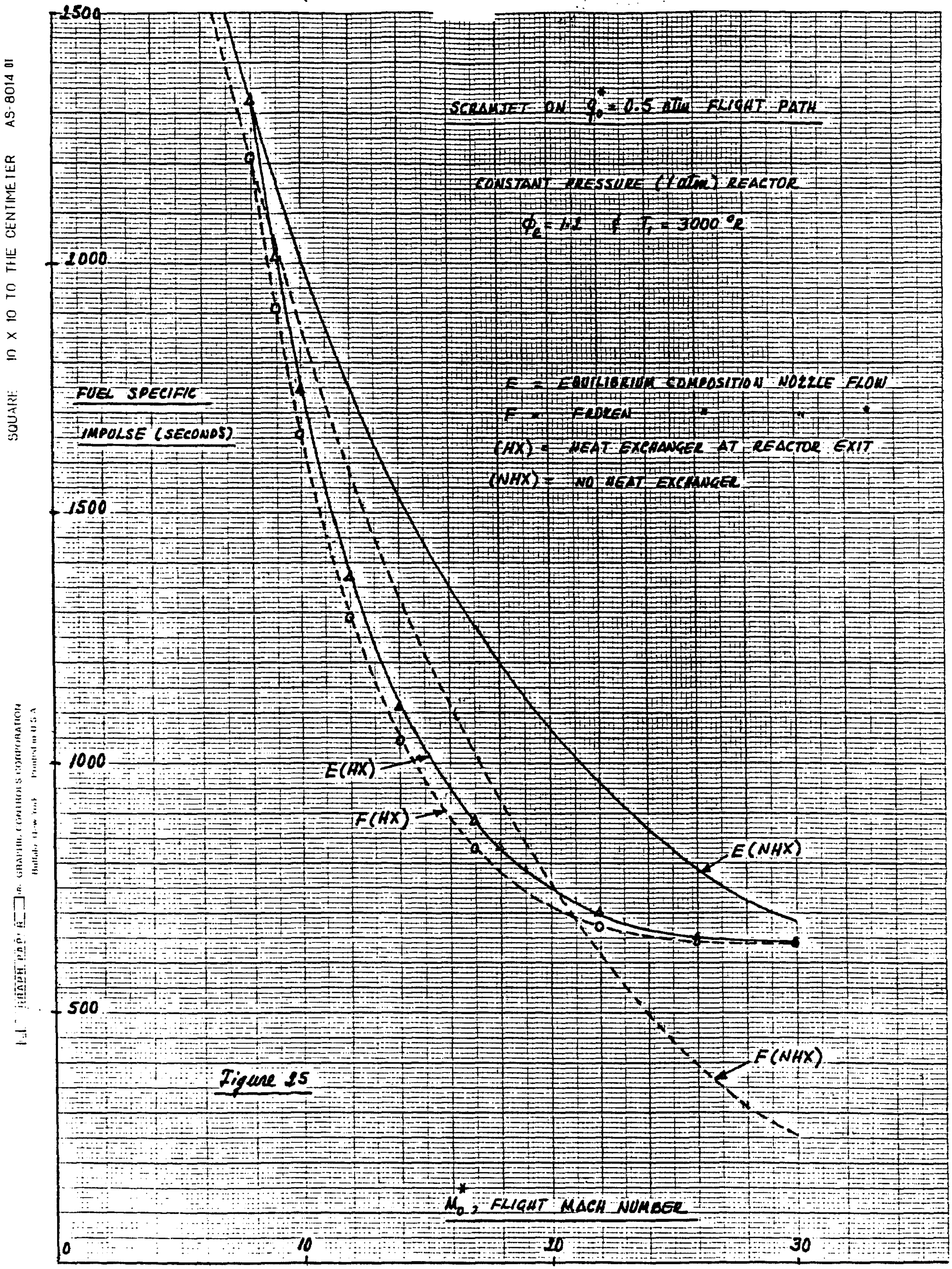

SCRAMJET ON $q_0^* = 0.5$ atm FLIGHT PATH
CONSTANT PRESSURE (1 atm) REACTOR
$\phi_e = 1.2$ & $T_1 = 3000\ °R$
E = EQUILIBRIUM COMPOSITION NOZZLE FLOW
F = FROZEN " " "
(HX) = HEAT EXCHANGER AT REACTOR EXIT
(NHX) = NO HEAT EXCHANGER
FUEL SPECIFIC IMPULSE (SECONDS)
2500
2000
1500
1000
500
E(HX)
F(HX)
E(NHX)
F(NHX)
M_0^*, FLIGHT MACH NUMBER
0
10
20
30
SQUARE 10 X 10 TO THE CENTIMETER AS-8014 01
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Figure 25

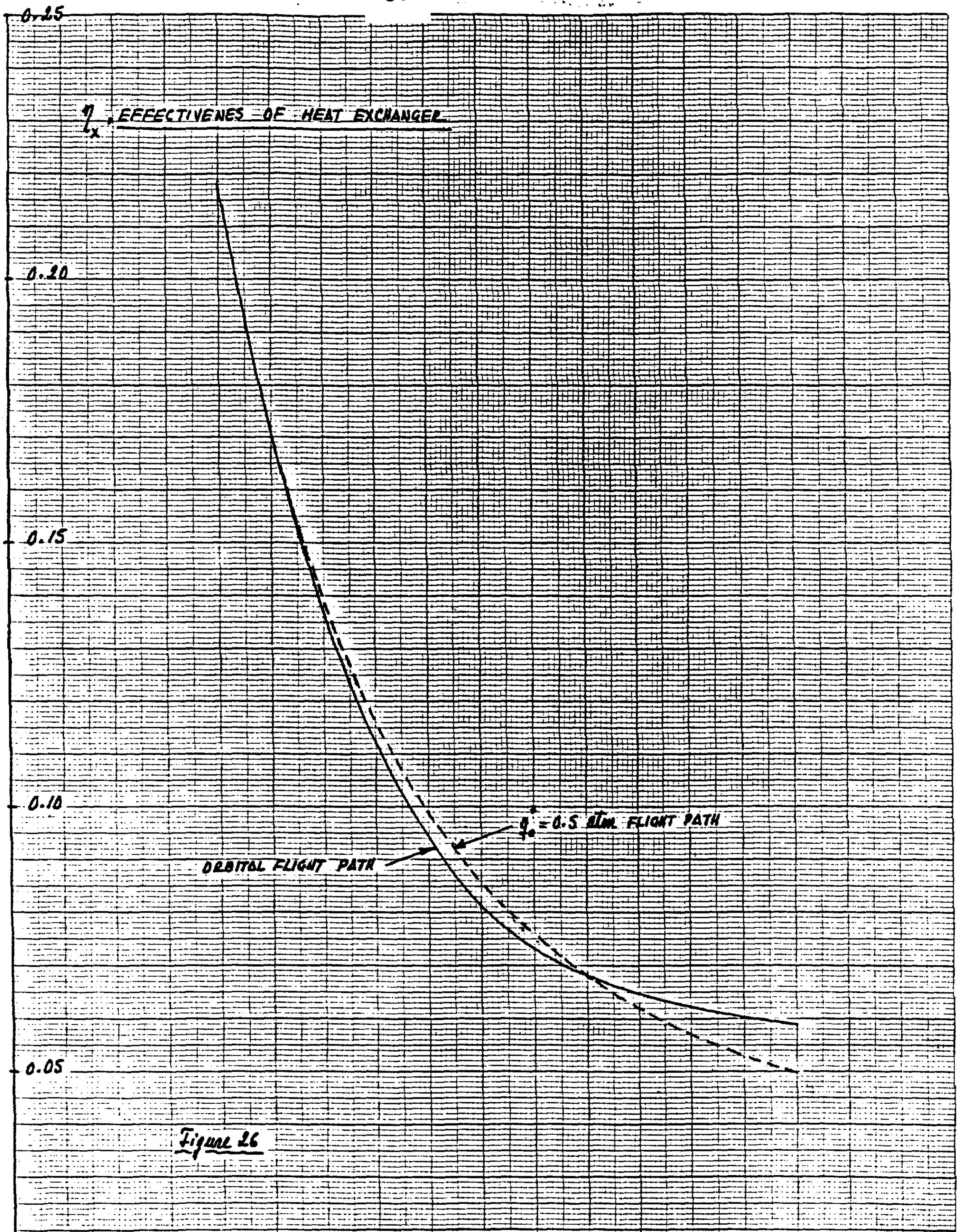

0.25
η_x, EFFECTIVENES OF HEAT EXCHANGER
0.20
0.15
0.10
$q_0^* = 0.5$ atm FLIGHT PATH
ORBITAL FLIGHT PATH
0.05
M_0^*, FLIGHT MACH NUMBER
0
10
20
30

Figure 26

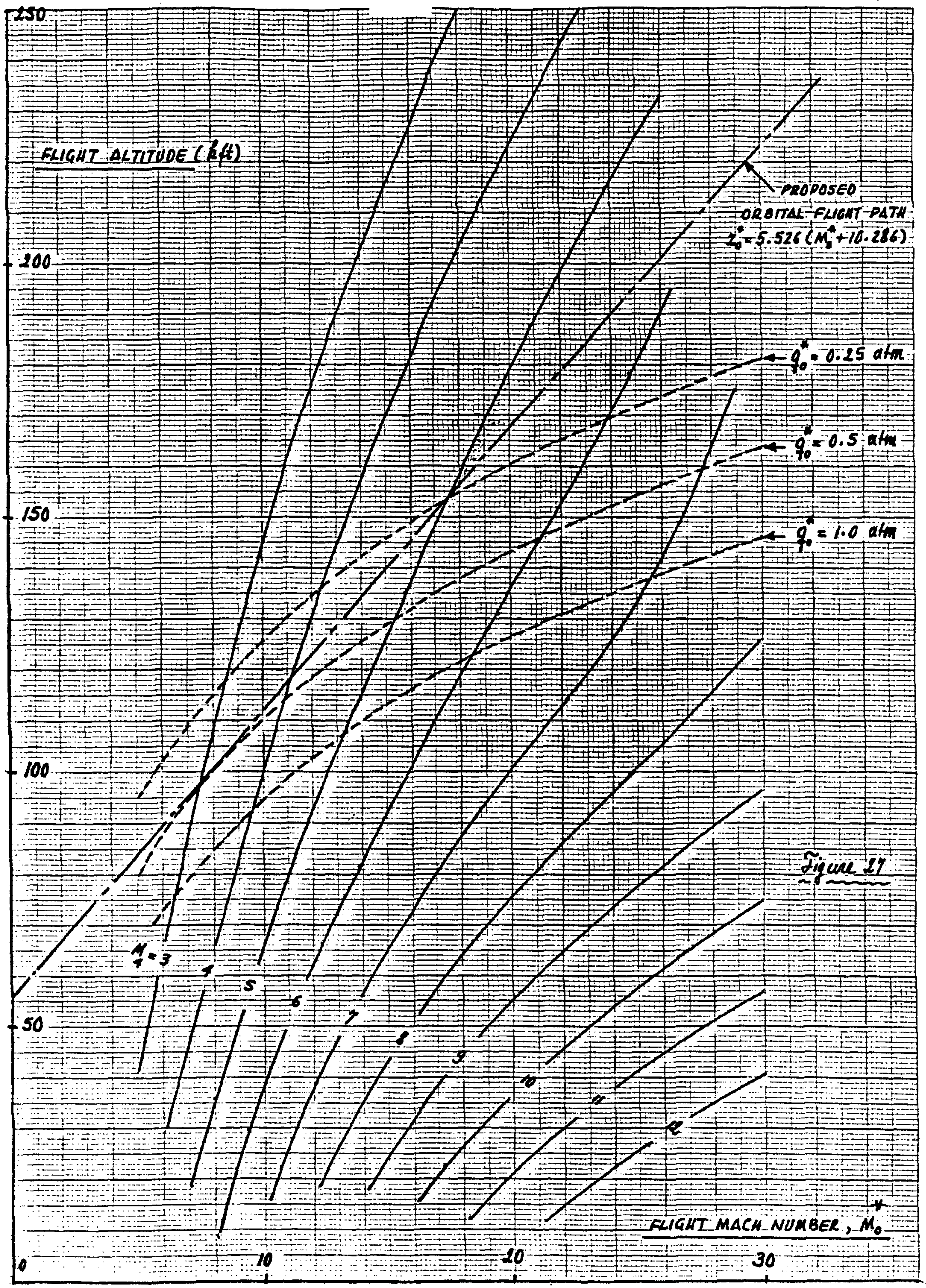
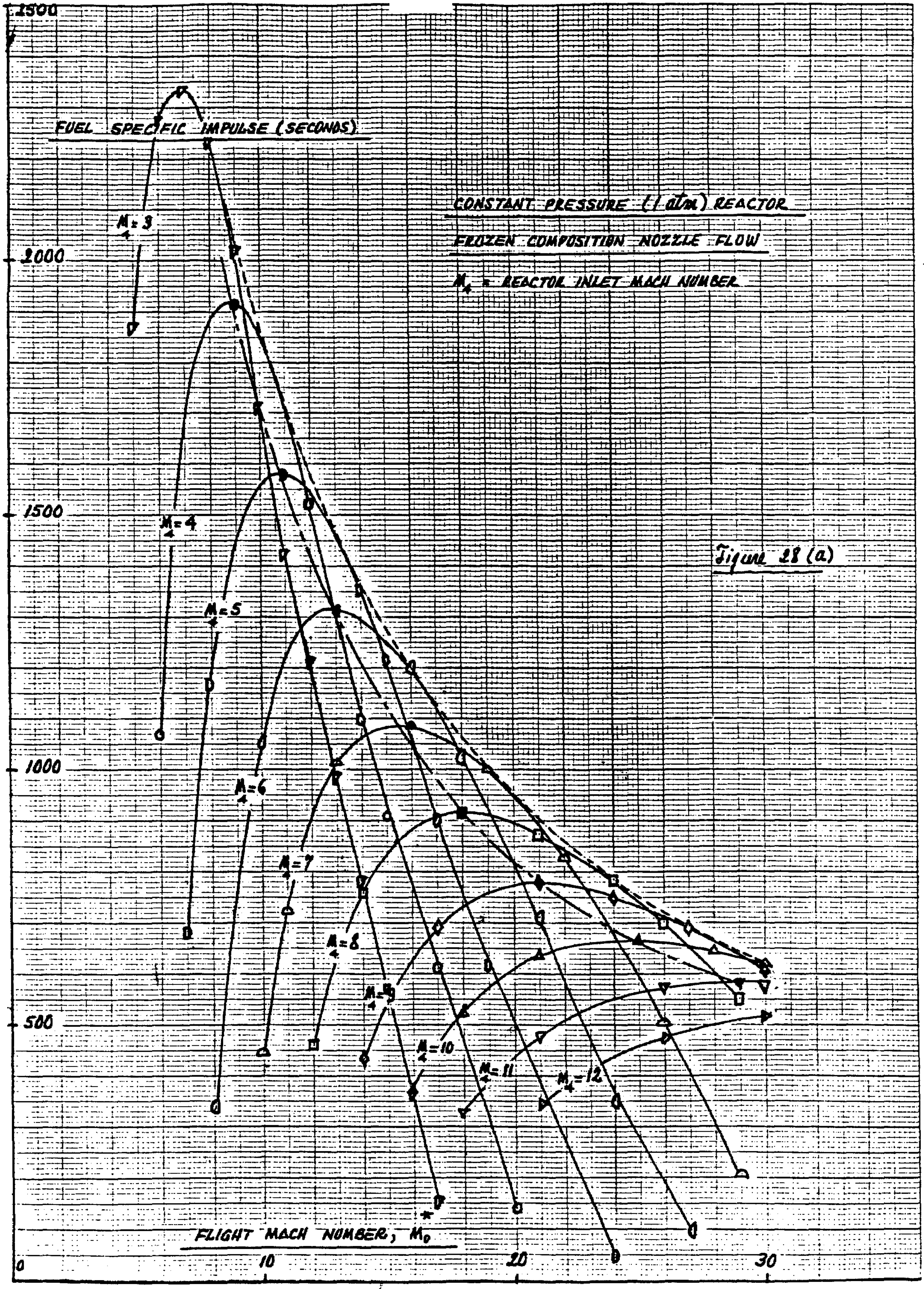
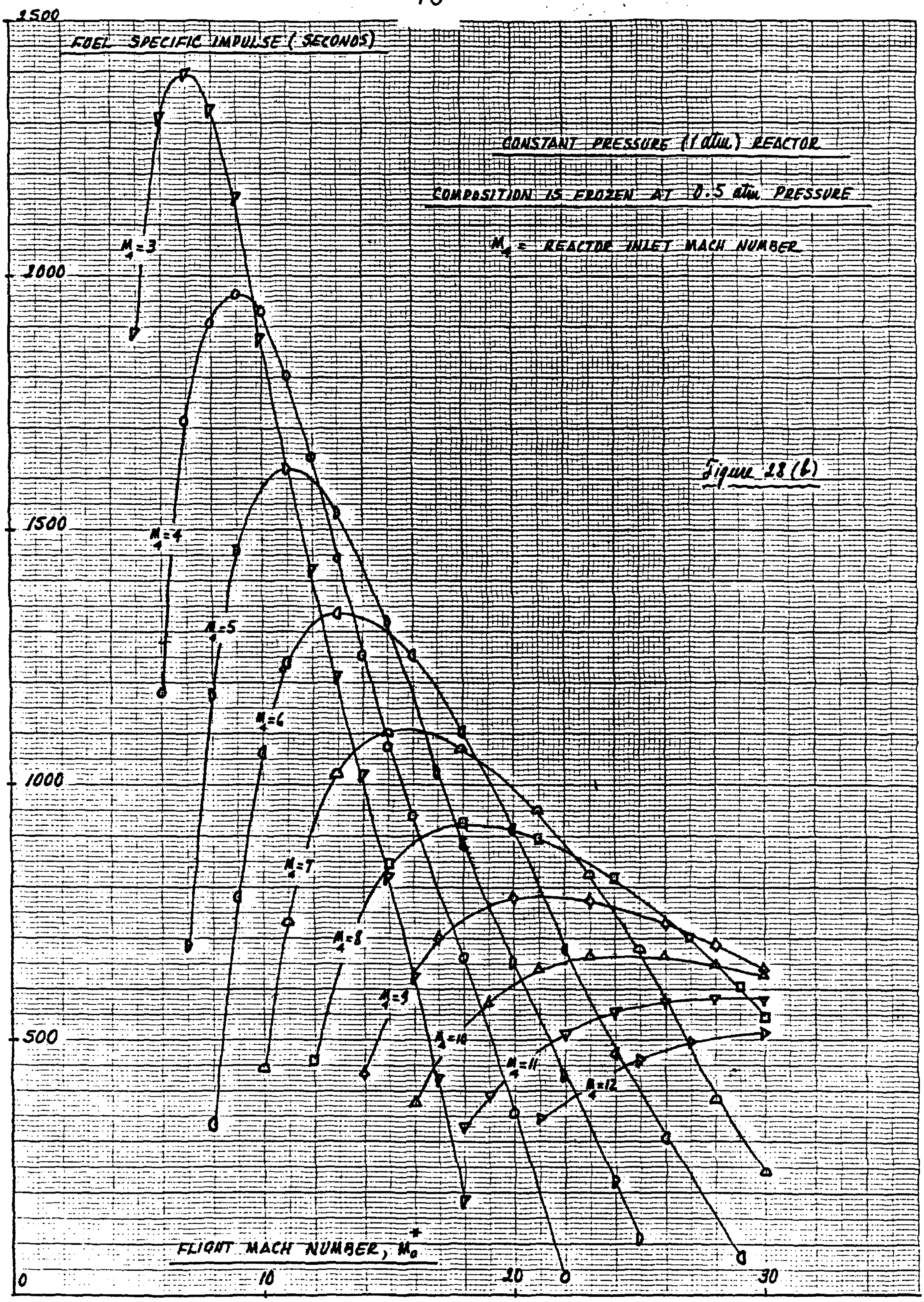

FLIGHT ALTITUDE (k ft)
250
200
150
100
50
0
10
20
30
PROPOSED ORBITAL FLIGHT PATH
$Z_0^* = 5.526(M_0^* + 10.286)$
$q_0^* = 0.25$ atm
$q_0^* = 0.5$ atm
$q_0^* = 1.0$ atm
$M_4 = 3$
4
5
6
7
8
9
10
11
12
FLIGHT MACH NUMBER, M_0^*

Figure 27



FUEL SPECIFIC IMPULSE (SECONDS)
CONSTANT PRESSURE (1 atm) REACTOR
FROZEN COMPOSITION NOZZLE FLOW
M_4 = REACTOR INLET MACH NUMBER
M_4 = 3
M_4 = 4
M_4 = 5
M_4 = 6
M_4 = 7
M_4 = 8
M_4 = 9
M_4 = 10
M_4 = 11
M_4 = 12
2500
2000
1500
1000
500
0
10
20
30
FLIGHT MACH NUMBER, M_0

Figure 28 (a)


2500
FUEL SPECIFIC IMPULSE (SECONDS)
CONSTANT PRESSURE (1 atm) REACTOR
COMPOSITION IS FROZEN AT 0.5 atm PRESSURE
M_4 = REACTOR INLET MACH NUMBER
Figure 28 (b)
2000
1500
1000
500
$M_4=3$
$M_4=4$
$M_4=5$
$M_4=6$
$M_4=7$
$M_4=8$
$M_4=9$
$M_4=10$
$M_4=11$
$M_4=12$
FLIGHT MACH NUMBER, M_0
0
10
20
30

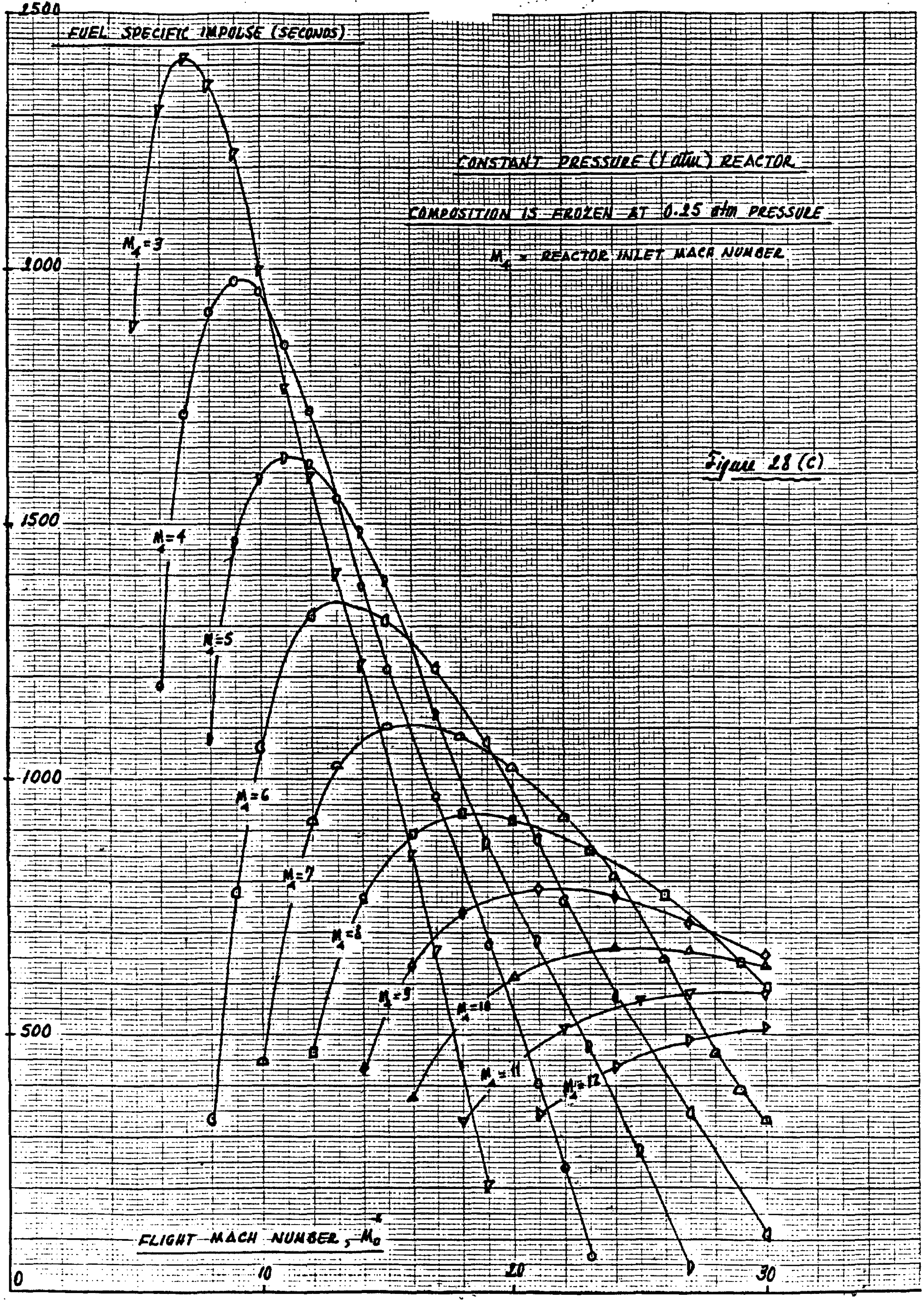
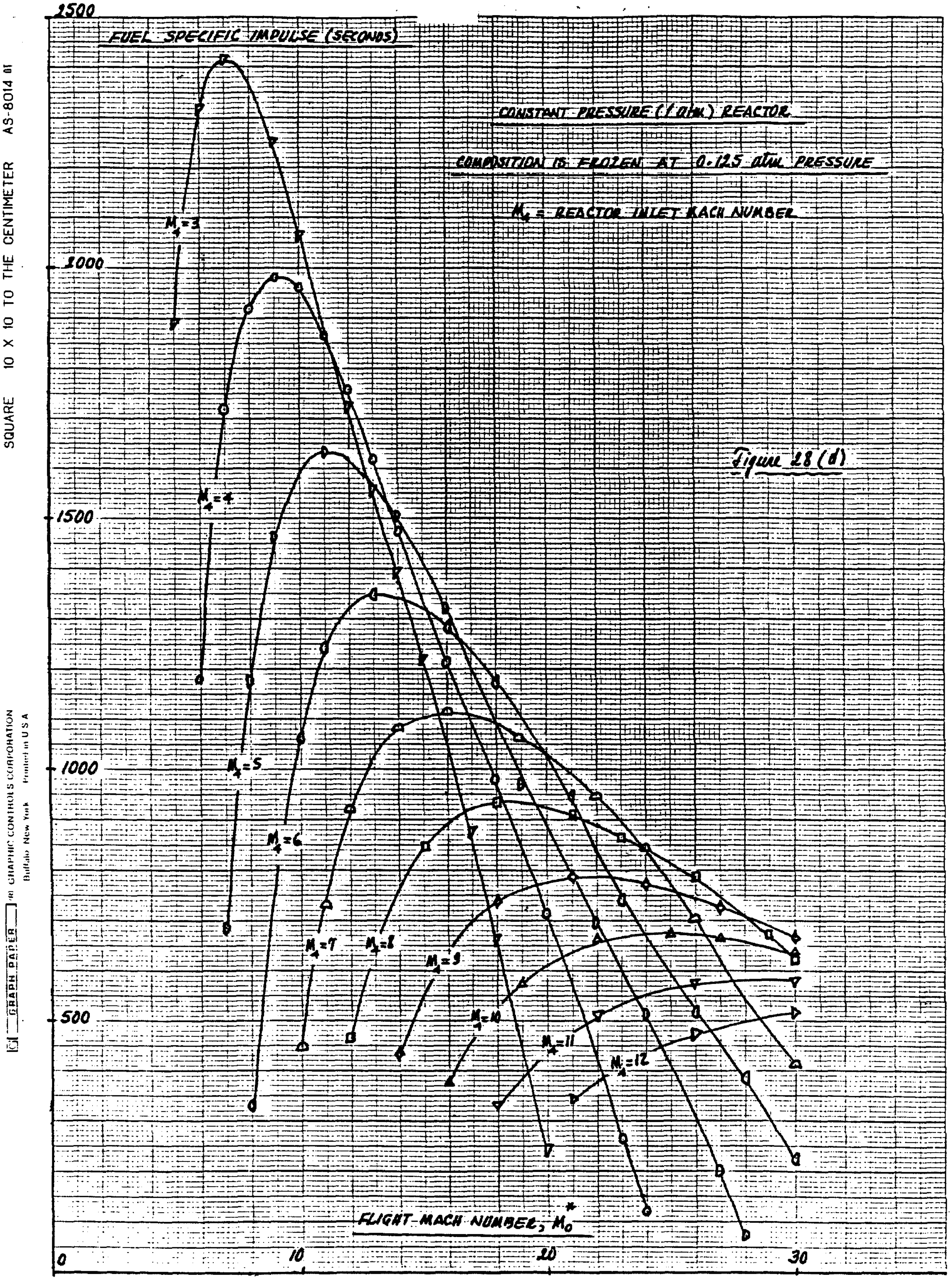

FUEL SPECIFIC IMPULSE (SECONDS)
2500
2000
1500
1000
500
0
10
20
30
FLIGHT MACH NUMBER, M_0
CONSTANT PRESSURE (1 atm) REACTOR
COMPOSITION IS FROZEN AT 0.25 atm PRESSURE
M_4 = REACTOR INLET MACH NUMBER
$M_4=3$
$M_4=4$
$M_4=5$
$M_4=6$
$M_4=7$
$M_4=8$
$M_4=9$
$M_4=10$
$M_4=11$
$M_4=12$

Figure 28 (c)


FUEL SPECIFIC IMPULSE (SECONDS)
CONSTANT PRESSURE (1 atm) REACTOR
COMPOSITION IS FROZEN AT 0.125 atm PRESSURE
M4 = REACTOR INLET MACH NUMBER
Figure 28 (d)
2500
2000
1500
1000
500
0
10
20
30
M4=3
M4=4
M4=5
M4=6
M4=7
M4=8
M4=9
M4=10
M4=11
M4=12
FLIGHT MACH NUMBER, M0*
SQUARE 10 X 10 TO THE CENTIMETER AS-8014 01
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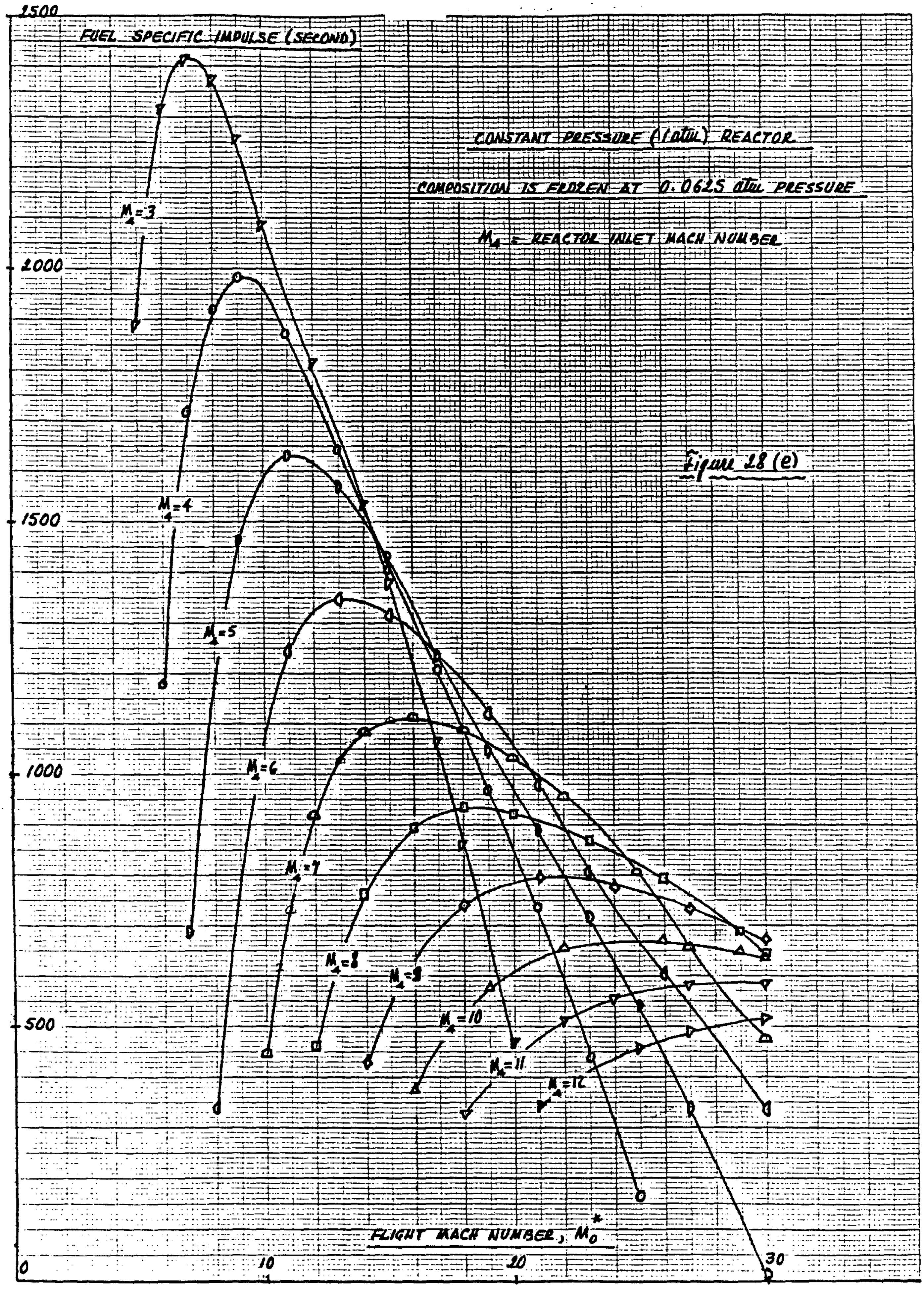
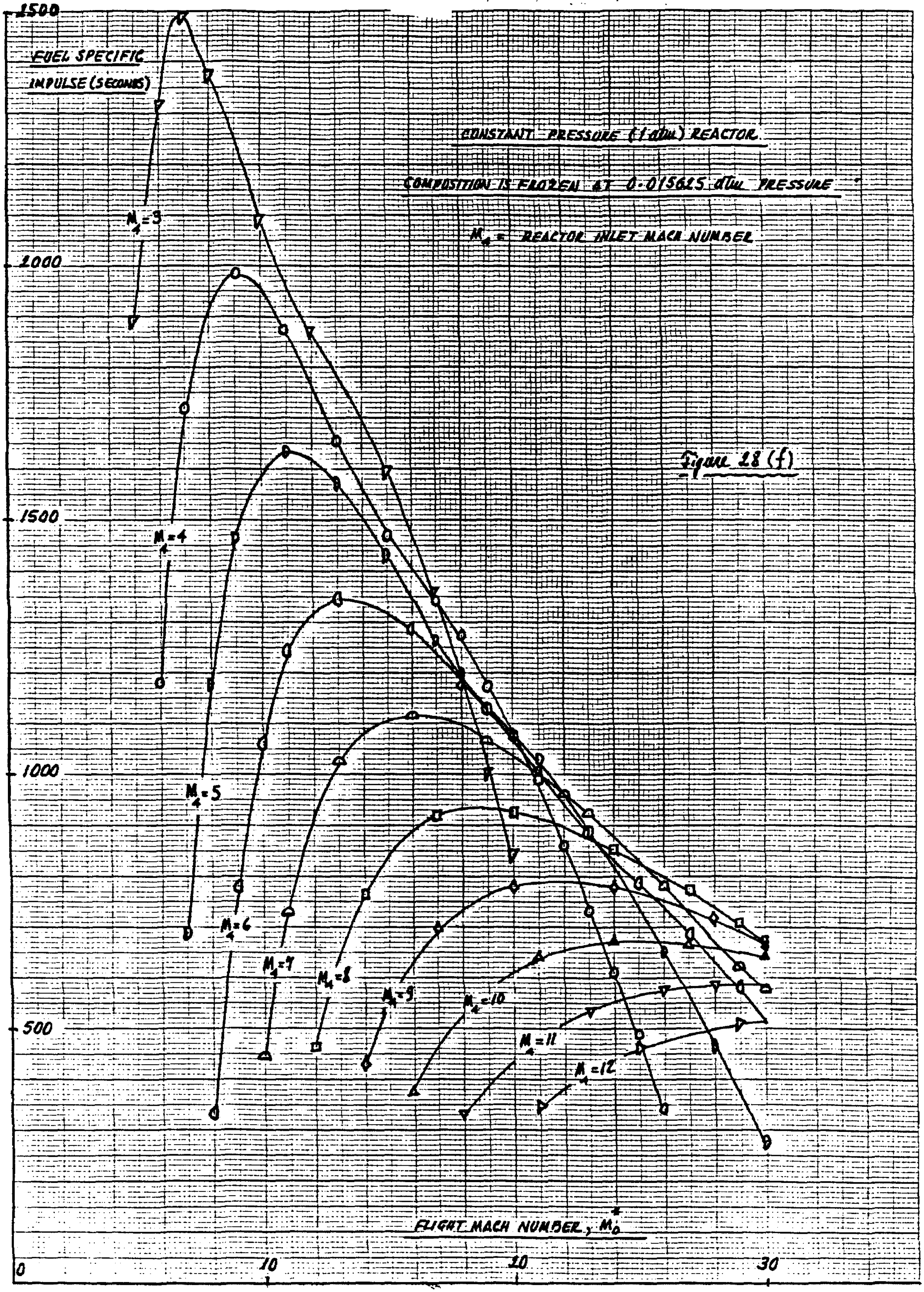
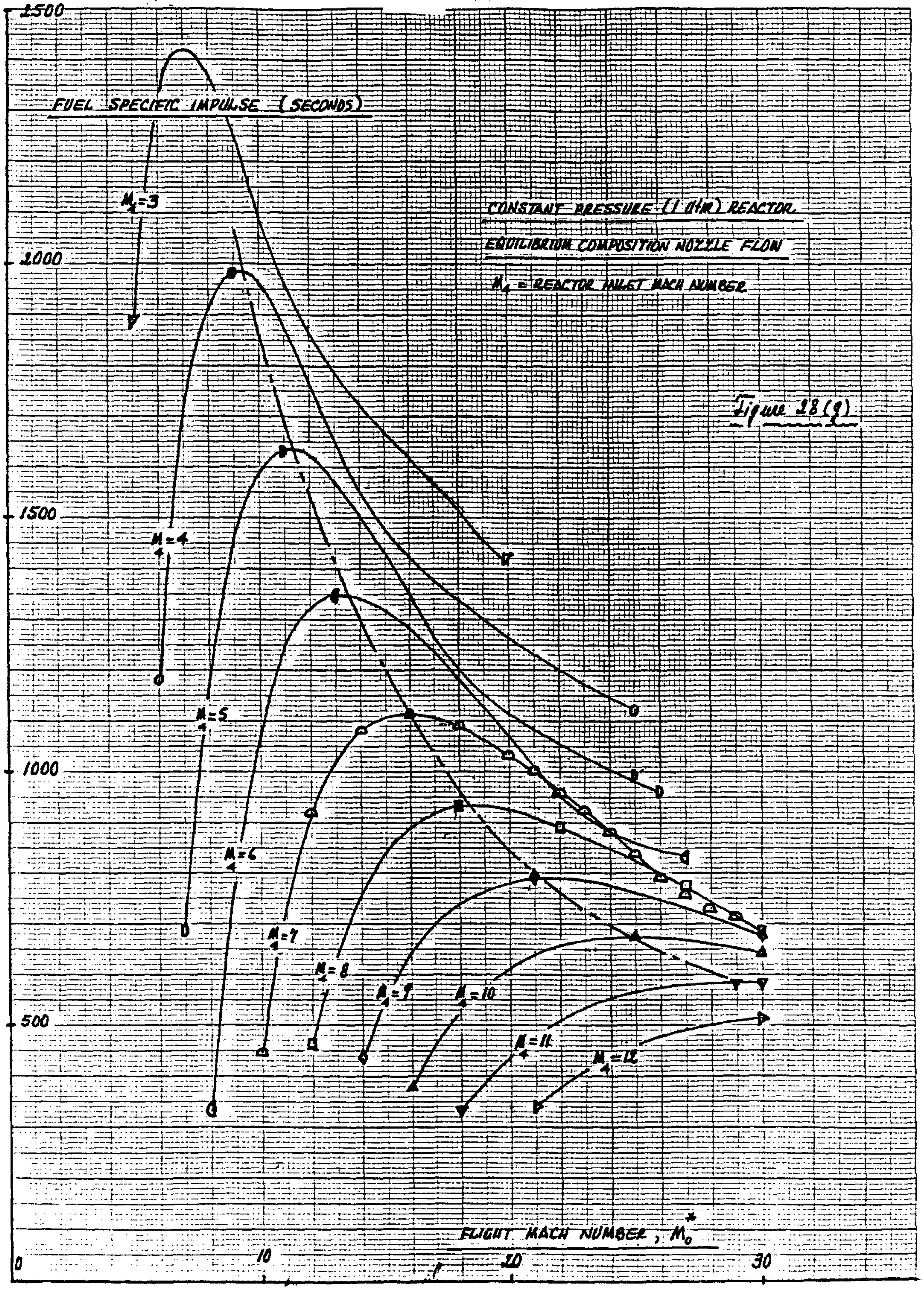
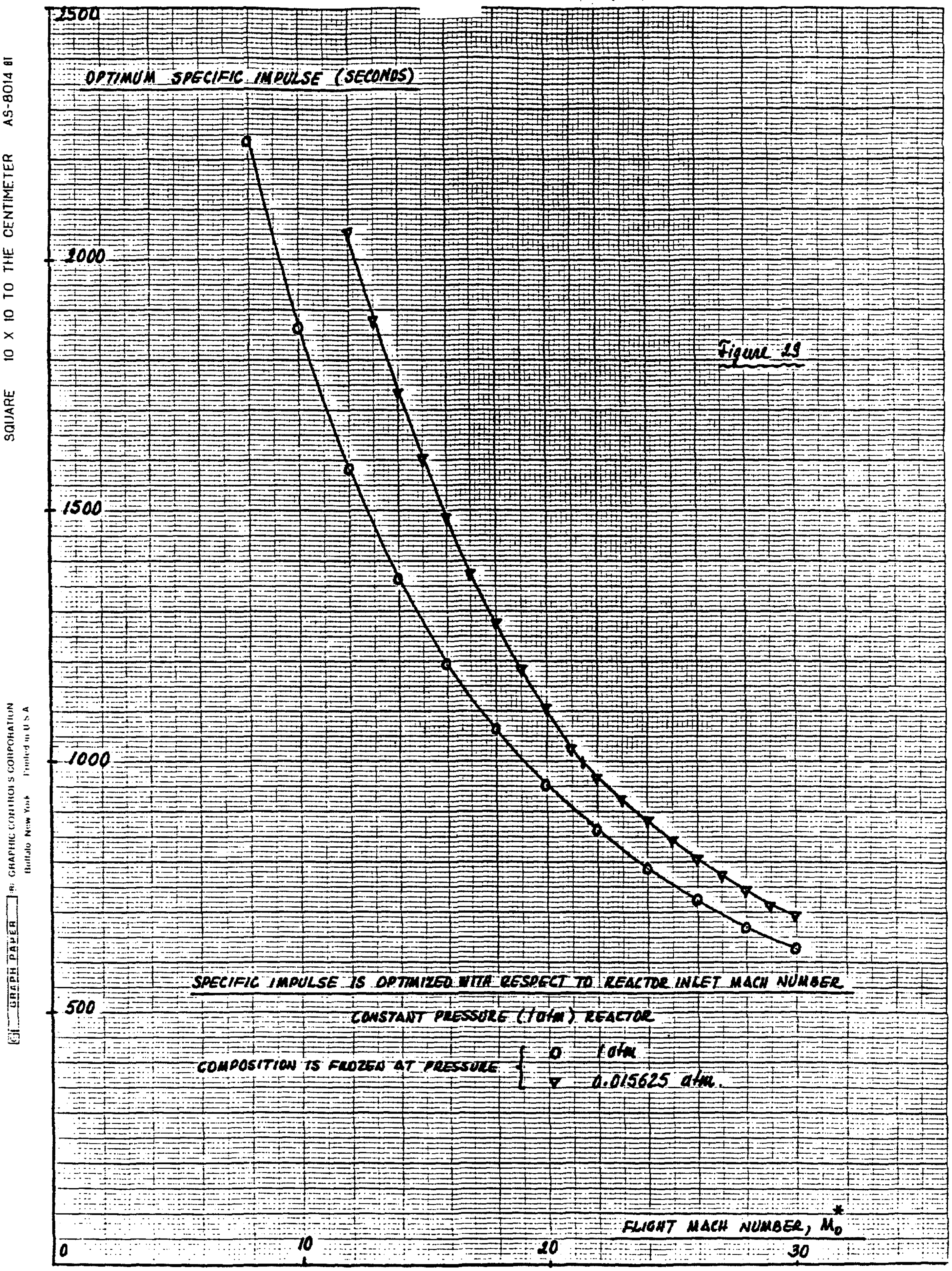

FUEL SPECIFIC IMPULSE (SECOND)
CONSTANT PRESSURE (1 atm) REACTOR
COMPOSITION IS FROZEN AT 0.0625 atm PRESSURE
M_4 = REACTOR INLET MACH NUMBER
Figure 28 (e)
$M_4=3$
$M_4=4$
$M_4=5$
$M_4=6$
$M_4=7$
$M_4=8$
$M_4=9$
$M_4=10$
$M_4=11$
$M_4=12$
FLIGHT MACH NUMBER, M_0^*
2500
2000
1500
1000
500
0
10
20
30


2500
FUEL SPECIFIC
IMPULSE (SECONDS)
CONSTANT PRESSURE (1 atm) REACTOR
COMPOSITION IS FROZEN AT 0.015625 atm PRESSURE
M_4 = REACTOR INLET MACH NUMBER
Figure 28 (f)
2000
1500
1000
500
0
M_4=3
M_4=4
M_4=5
M_4=6
M_4=7
M_4=8
M_4=9
M_4=10
M_4=11
M_4=12
10
20
30
FLIGHT MACH NUMBER, M_0^*


2500
2000
1500
1000
500
0
10
20
30
FUEL SPECIFIC IMPULSE (SECONDS)
CONSTANT PRESSURE (1 ATM) REACTOR
EQUILIBRIUM COMPOSITION NOZZLE FLOW
M_4 = REACTOR INLET MACH NUMBER
Figure 28(g)
$M_4=3$
$M_4=4$
$M_4=5$
$M_4=6$
$M_4=7$
$M_4=8$
$M_4=9$
$M_4=10$
$M_4=11$
$M_4=12$
FLIGHT MACH NUMBER, M_0^*



OPTIMUM SPECIFIC IMPULSE (SECONDS)
2500
2000
1500
1000
500
Figure 29
SPECIFIC IMPULSE IS OPTIMIZED WITH RESPECT TO REACTOR INLET MACH NUMBER
CONSTANT PRESSURE (1 atm) REACTOR
COMPOSITION IS FROZEN AT PRESSURE
o 1 atm
▽ 0.015625 atm.
FLIGHT MACH NUMBER, M0*
0
10
20
30
SQUARE 10 X 10 TO THE CENTIMETER AS-8014 81
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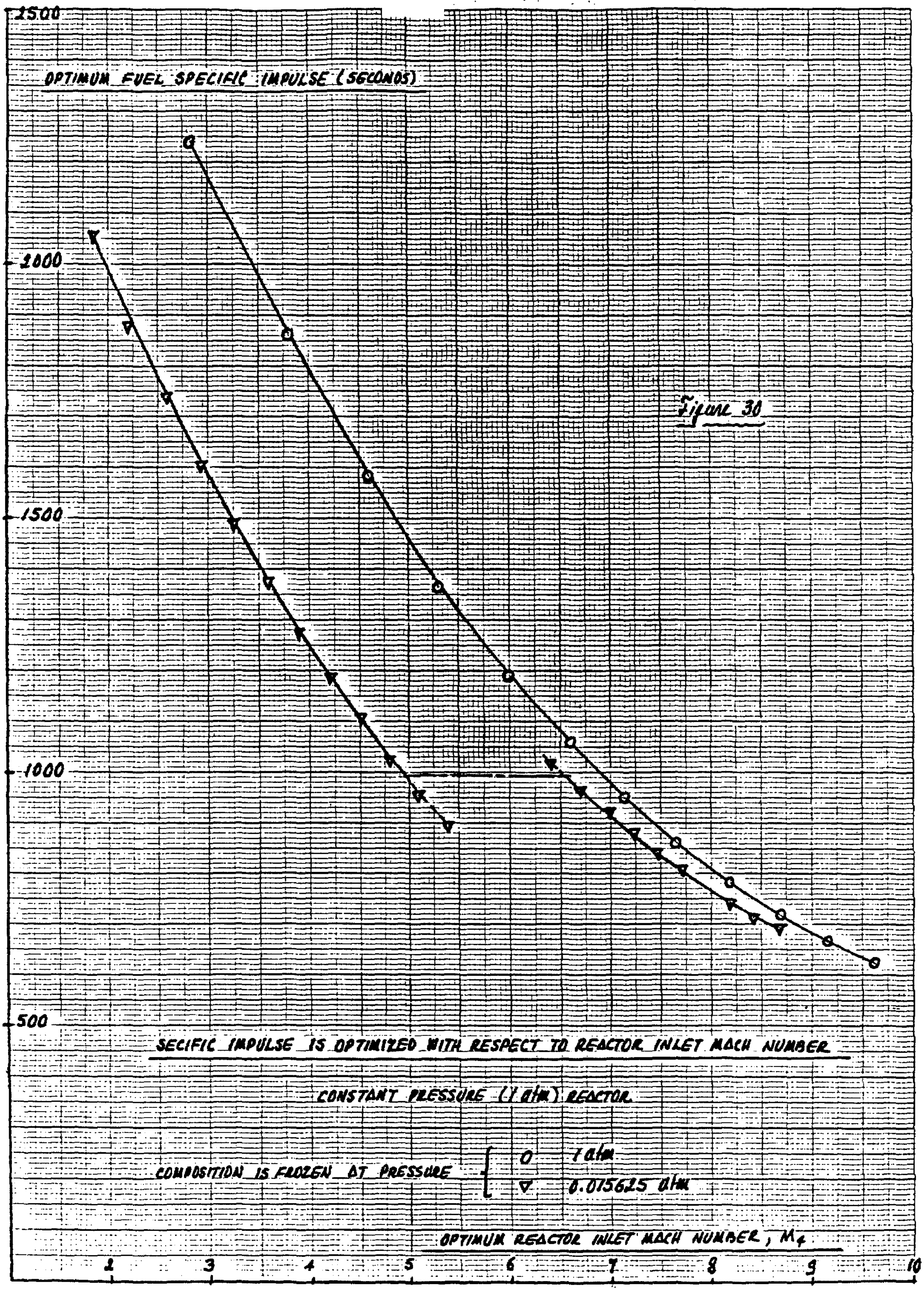

OPTIMUM FUEL SPECIFIC IMPULSE (SECONDS)
2500
2000
1500
1000
500
Figure 30
SECIFIC IMPULSE IS OPTIMIZED WITH RESPECT TO REACTOR INLET MACH NUMBER
CONSTANT PRESSURE (1 atm) REACTOR
COMPOSITION IS FROZEN AT PRESSURE
O 1 atm
▽ 0.015625 atm
OPTIMUM REACTOR INLET MACH NUMBER, M_4
2
3
4
5
6
7
8
9
10

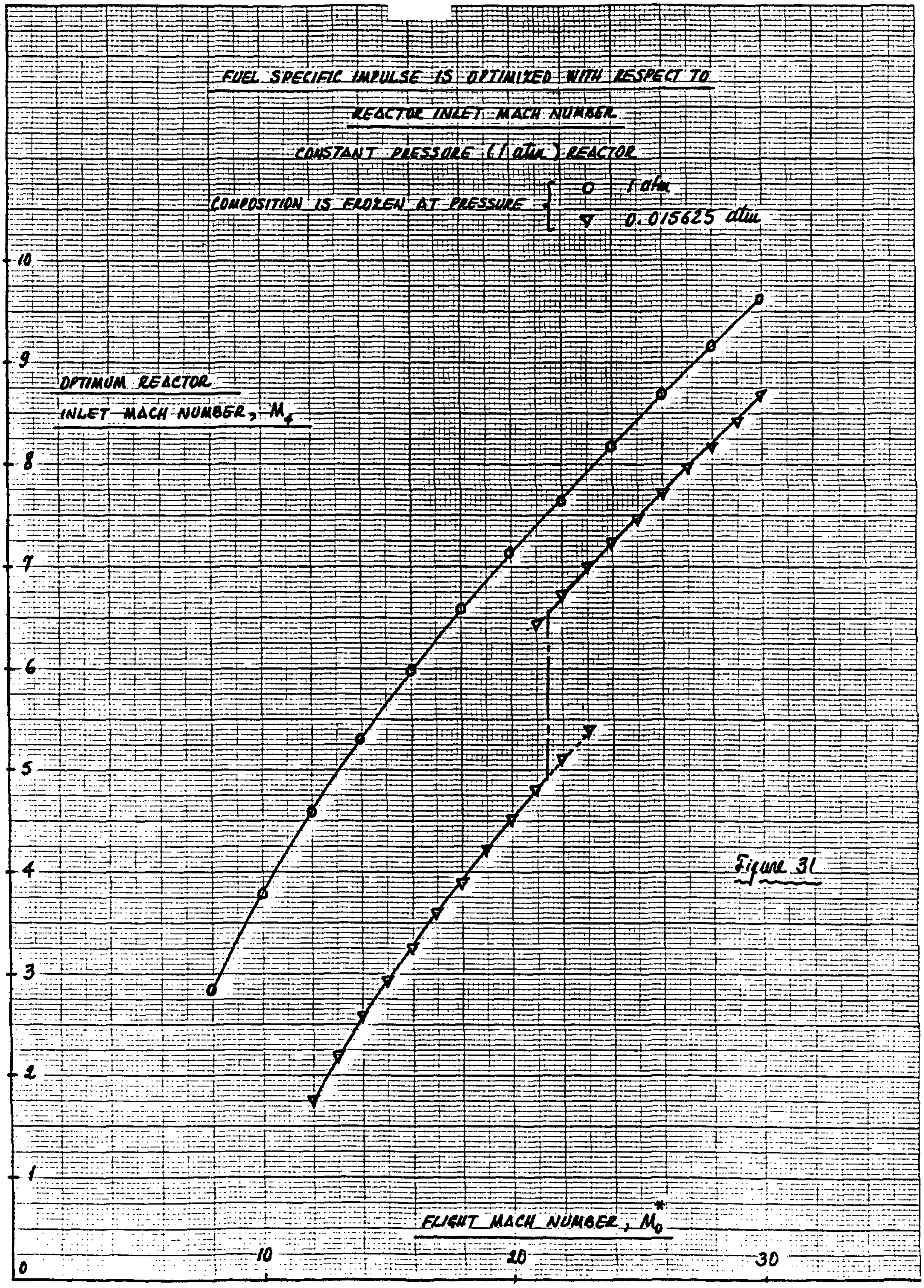

FUEL SPECIFIC IMPULSE IS OPTIMIZED WITH RESPECT TO
REACTOR INLET MACH NUMBER
CONSTANT PRESSURE (1 atm) REACTOR
COMPOSITION IS FROZEN AT PRESSURE
o 1 atm
▽ 0.015625 atm
OPTIMUM REACTOR
INLET MACH NUMBER, M_4
FLIGHT MACH NUMBER, M_0^*
0
1
2
3
4
5
6
7
8
9
10
10
20
30

Figure 31

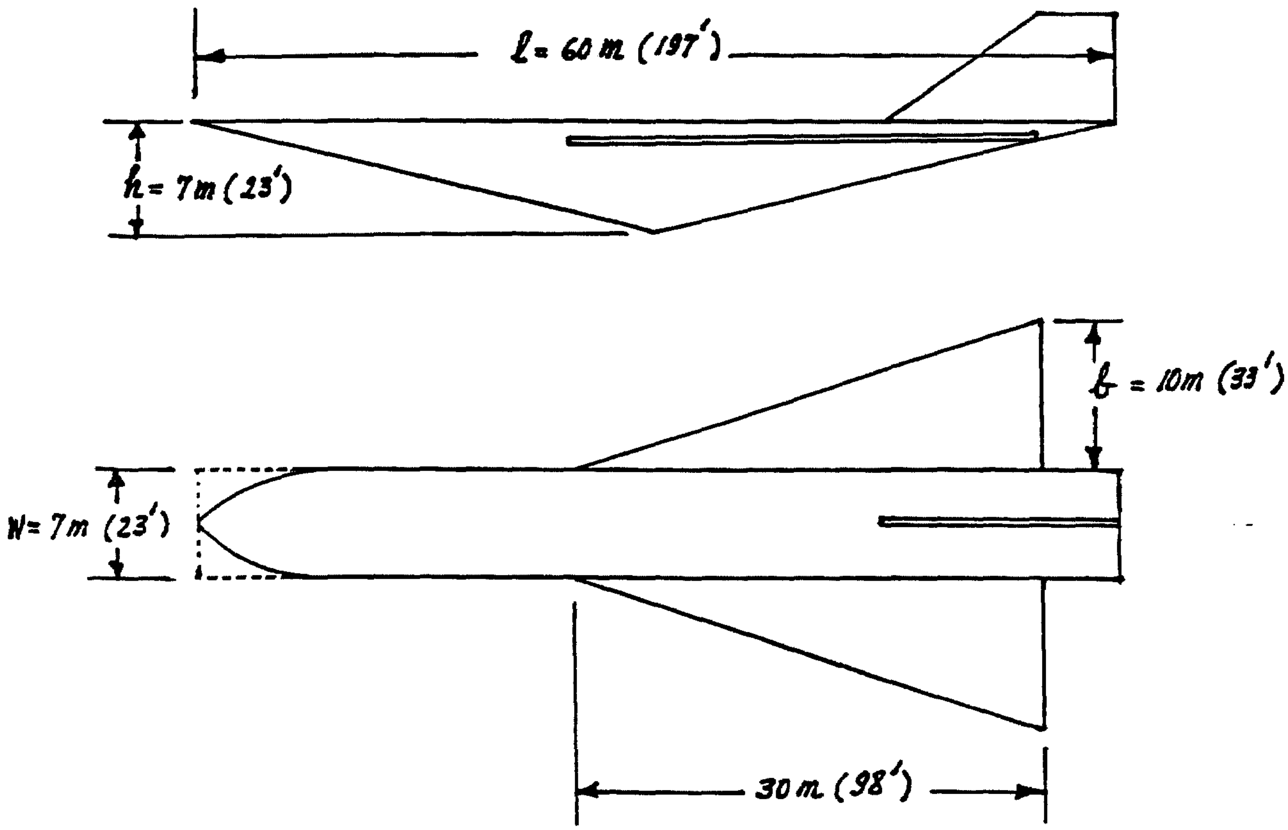


Figure 32

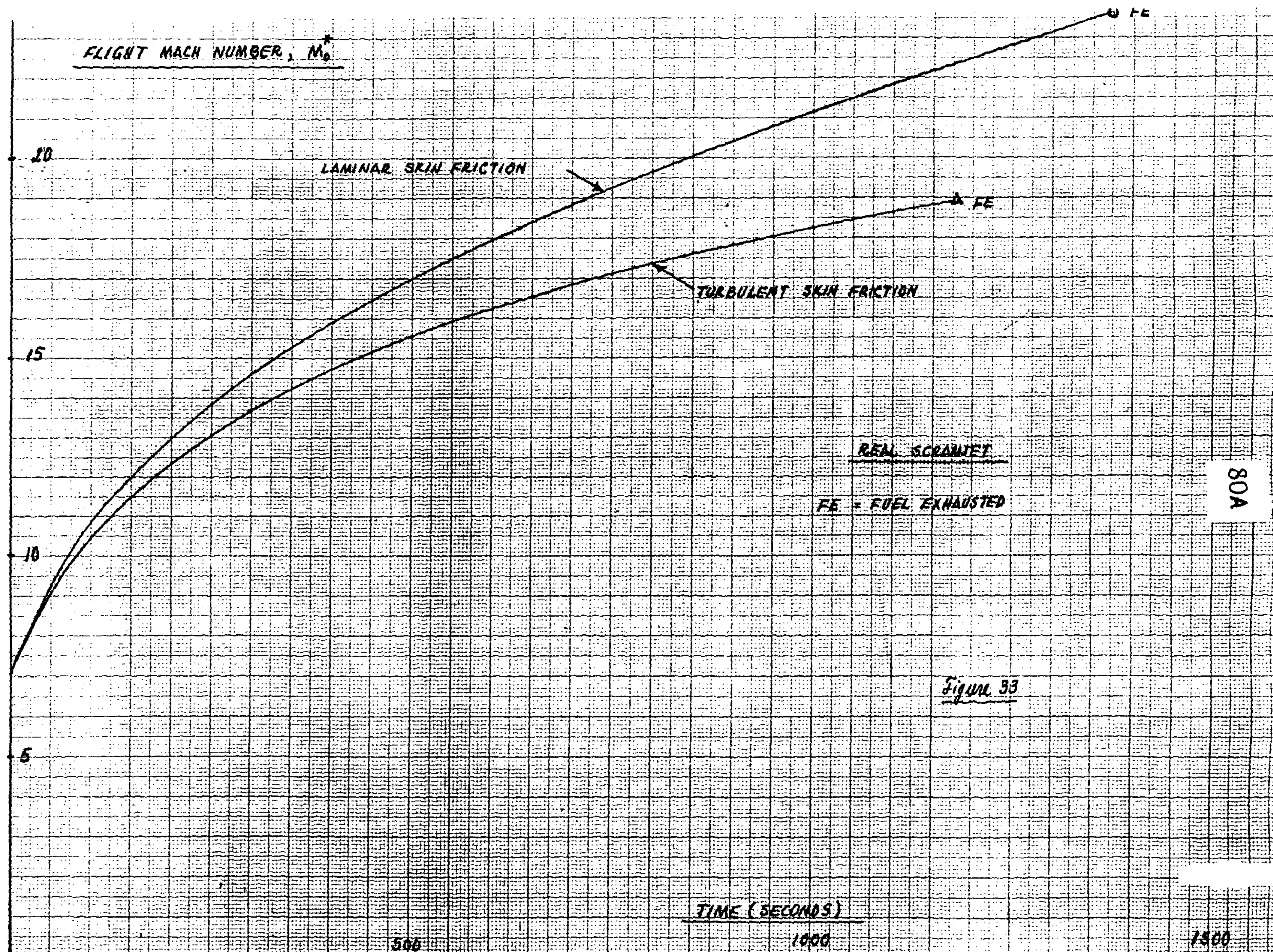

FLIGHT MACH NUMBER, M_0^*
20
15
10
5
LAMINAR SKIN FRICTION
TURBULENT SKIN FRICTION
FE
REAL SCRAMJET
FE = FUEL EXHAUSTED
TIME (SECONDS)
500
1000
1500

Figure 33

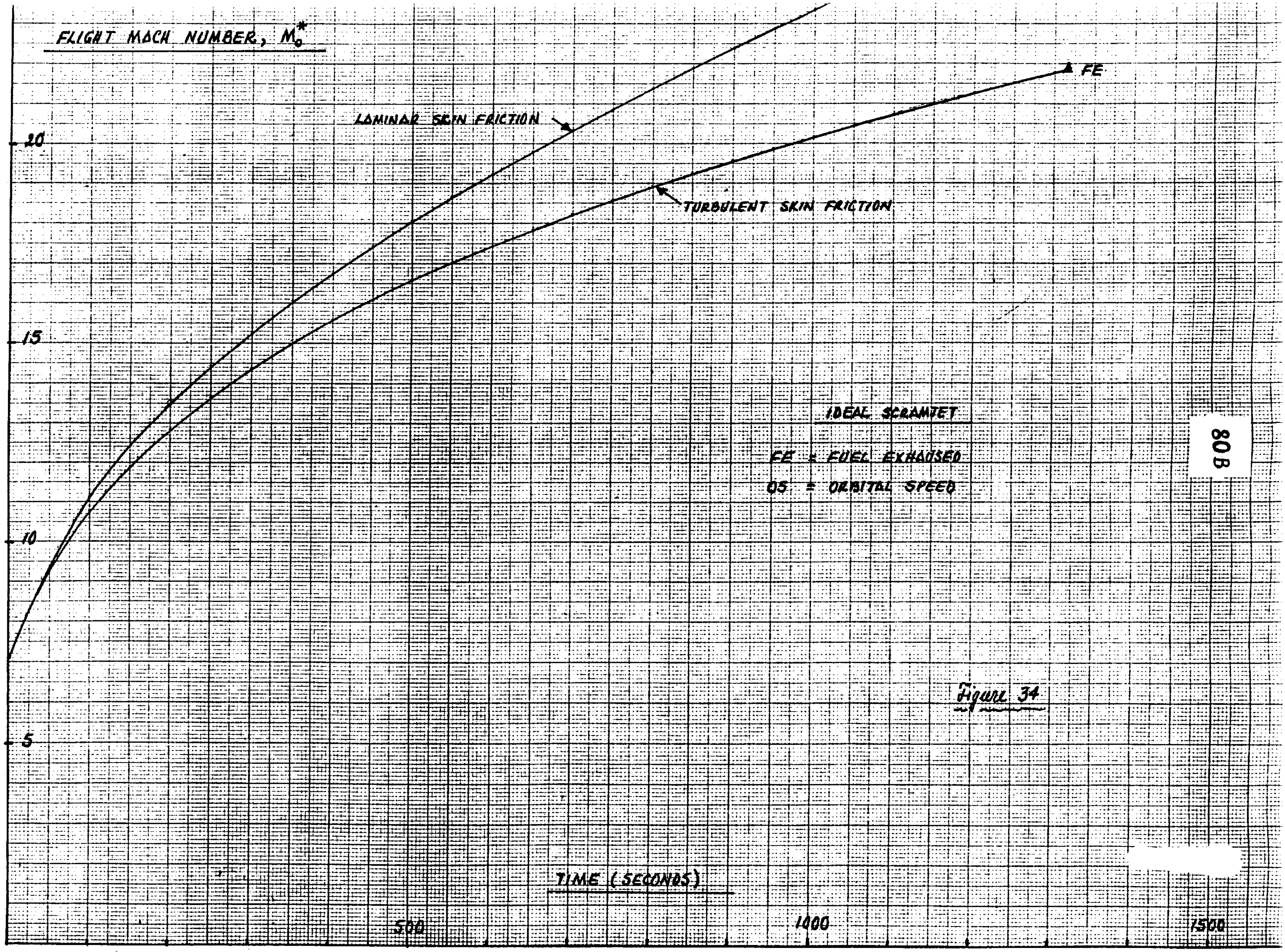

FLIGHT MACH NUMBER, M_0^*
LAMINAR SKIN FRICTION
TURBULENT SKIN FRICTION
FE
IDEAL SCRAMJET
FE = FUEL EXHAUSTED
OS = ORBITAL SPEED
20
15
10
5
TIME (SECONDS)
500
1000
1500

Figure 34

APPENDIX A
ON THE CHEMICAL EQUILIBRIUM MOLE FRACTION DETERMINATION

Introduction:

When chemical reactions are near equilibrium, partial pressures of various chemical species are governed by the law of mass action, which states that for any chemically interacting compounds A,B,...,L,M,... that satisfy the stoichiometric equation:

$$\nu_A' A + \nu_B' + \cdots \rightleftharpoons \nu_L L + \nu_M M + \cdots \qquad \text{(A-1)}$$

where $\nu_A', \nu_B', \ldots, \nu_L, \ldots$ are number of moles of compounds $A, B, \ldots, L, \ldots$ respectively, then there is a function K_p of temperature, alone such that

$$K_p(T) = \frac{(\bar{p}_L)^{\nu_L} (\bar{p}_M)^{\nu_M} \cdots}{(\bar{p}_A)^{\nu_A'} (\bar{p}_B)^{\nu_B'} \cdots} \qquad \text{(A-2)}$$

where $K_p(T)$ is called the partial pressure equilibrium constant of the reaction and $\bar{p}_A, \ldots, \bar{p}_L, \ldots$ are partial pressures of compounds $A, \ldots, L, \ldots$ normalized with respect to a standard pressure, p^0 .

It is convenient to use the mole fraction equilibrium constant, $K_y(T, p)$ instead of $K_p(T)$, where

$$K_y(T, p) = \frac{y_L^{\nu_L} \cdot y_M^{\nu_L} \cdots}{y_A^{\nu_A'} \cdot y_B^{\nu_B'}} \qquad \text{(A-3)}$$

where $y_A, \ldots, y_L, \ldots$ are mole fractions of compounds $A, \ldots, L, \ldots$.

Normalized partial pressures of compounds are related to mole fractions of compounds, that is to say, $\bar{p}_A = y_A \bar{p}, \ldots$ with $\bar{p}$ denoting normalized mixture pressure, consequently $K_y(T,p)$ is related to K_p (T) in the following manner:

$$K_y(T, p) = K_p(T) \cdot (\bar{p})^{[(\nu_A' + \nu_B' + \cdots) - (\nu_L + \nu_M + \cdots)]} \qquad \text{(A-4)}$$

JANAF Thermochemical Tables (reference [2]) list formation equilibrium constants of various compounds, as they are formed from their elements, as a function of temperature and at standard pressure, $p^0 = 1$ bar. These formation equilibrium constants denoted by K_f are in fact partial pressure equilibrium constants as specified in equation (A-2).

In this analysis nine compounds are considered, namely O, H, OH, H_2O, N, NO, HO_2, NO_2 and N_2O . Below are reactions that form these compounds from elements O_2, H_2, and N_2 as indicated in equation (A-1). For each compound forms of equation (A-3) are presented.

$$\left.\begin{array}{llll}
1) & \frac{1}{2} O_2 \rightleftarrows O & , \; y_O = K_1 \, y_{O_2}^{1/2} & \text{where} \quad K_1 = (K_f)_O \, (\bar{p})^{-1/2} \\
2) & \frac{1}{2} H_2 \rightleftarrows H & , \; y_H = K_2 \, y_{H_2}^{1/2} & \text{where} \quad K_2 = (K_f)_H \, (\bar{p})^{-1/2} \\
3) & \frac{1}{2} H_2 + \frac{1}{2} O_2 \rightleftarrows OH & , \; y_{OH} = K_3 \, y_{H_2}^{1/2} \, y_{O_2}^{1/2} & \text{where} \quad K_3 = (K_f)_{OH} \\
4) & H_2 + \frac{1}{2} O_2 \rightleftarrows H_2O & , \; y_{H_2O} = K_4 \, y_{H_2} \, y_{O_2}^{1/2} & \text{where} \quad K_4 = (K_f)_{H_2O} (\bar{p})^{1/2} \\
5) & \frac{1}{2} N_2 \rightleftarrows N & , \; y_N = K_5 \, y_{N_2}^{1/2} & \text{where} \quad K_5 = (K_f)_N \, (\bar{p})^{-1/2} \\
6) & \frac{1}{2} N_2 + \frac{1}{2} O_2 \rightleftarrows NO & , \; y_{NO} = K_6 \, y_{N_2}^{1/2} \, y_{O_2}^{1/2} & \text{where} \quad K_6 = (K_f)_{NO} \\
7) & \frac{1}{2} H_2 + O_2 \rightleftarrows HO_2 & , \; y_{HO_2} = K_7 \, y_{H_2}^{1/2} \, y_{O_2} & \text{where} \quad K_7 = (K_f)_{HO_2} (\bar{p})^{1/2} \\
8) & \frac{1}{2} N_2 + O_2 \rightleftarrows NO_2 & , \; y_{NO_2} = K_8 \, y_{N_2}^{1/2} \, y_{O_2} & \text{where} \quad K_8 = (K_f)_{NO_2} (\bar{p})^{1/2} \\
9) & \frac{1}{2} O_2 + N_2 \rightleftarrows N_2O & , \; y_{N_2O} = K_9 \, y_{N_2} \, y_{O_2}^{1/2} & \text{where} \quad K_9 = (K_f)_{N_2O} (\bar{p})^{1/2}
\end{array}\right\} \text{(A-5)}$$

<u>Fuel Dissociation or Recombination</u>: Fuel is assumed to have only two species: H and H_2 at a specified temperature, T and pressure, p. The objective is to determine their respective mole fractions y_H and y_{H_2} . For this purpose a known quantity, say n_T' moles, of fuel is considered to be at a convenient temperature, T' and pressure, p' such that mole fractions y_H' and y_{H_2}' of species H and H_2 respectively can be determined easily (normally $T' = 298.15\,K$ and $p' = 1$ bar). Next, the same quantity of fuel is brought to desired temperature, T and pressure, p as if the following unidirectional chemical reaction is taken place:

$$n_T' \left(y_H' H + y_{H_2}' H_2 \right) \rightarrow n_T \left(y_H H + y_{H_2} H_2 \right) \qquad \text{(A-6)}$$

where n_T is the number of moles of fuel at temperature, T and pressure, p. By definition,

$$y_H + y_{H_2} = 1 \qquad \text{(A-7)}$$

and conservation of mass of H requires that

$$n_T' \left(y_H' + 2\, y_{H_2}' \right) = n_T \left(y_H + 2\, y_{H_2} \right) \tag{A-8}$$

By substituting $y_H = K_2\, y_{H_2}^{1/2}$ in equation (A-7) and then solving for y_{H_2} one obtains

$$y_{H_2}^{1/2} = -\frac{K_2}{2} + \left\{ 1 + \left(\frac{K_2}{2}\right)^2 \right\}^{1/2} \tag{A-9}$$

Finally, if need be, n_T is determined by using equation (A-8).

<u>Air Dissociation or Recombination</u>: Air is assumed to have following species: $O, N, NO, NO_2, N_2O, O_2, N_2$, and Ar.

The unidirectional chemical reaction for this case is:

$$n_T' \left(y_O' O + y_N' N + y_{NO}' NO + y_{NO_2}' NO_2 + y_{N_2O}' N_2O + y_{O_2}' O_2 + y_{N_2}' N_2 + y_{Ar}' Ar \right) \longrightarrow$$
$$n_T \left(y_O O + y_N N + y_{NO} NO + y_{NO_2} NO_2 + y_{N_2O} N_2O + y_{O_2} O_2 + y_{N_2} N_2 + y_{Ar} Ar \right) \tag{A-10}$$

With known values of $n_T', y_O', \cdots y_{Ar}'$, the objective is to determine $n_T, y_O, \cdots, y_{Ar}$ at specified temperature, T and pressure, p. By definition,

$$y_O + y_N + y_{NO} + y_{NO_2} + y_{N_2O} + y_{O_2} + y_{N_2} + y_{Ar} = 1 \tag{A-11}$$

Conservation of masses of O, N, and Ar yield following relations respectively

$$n_T' \left(y_O' + y_{NO}' + 2 y_{NO_2}' + y_{N_2O}' + 2 y_O' \right) = n_T \left(y_O + y_{NO} + 2 y_{NO_2} + y_{N_2O} + 2 y_{O_2} \right) \tag{A-12}$$

$$n_T' \left(y_N' + y_{NO}' + y_{NO_2}' + 2 y_{N_2O}' + 2 y_{N_2}' \right) = n_T \left(y_N + y_{NO} + y_{NO_2} + 2 y_{N_2O} + 2 y_{N_2} \right) \tag{A-13}$$

and

$$n_T' y_{Ar}' = n_T y_{Ar} \tag{A-14}$$

By letting $U = y_{O_2}^{1/2}$ and $W = y_{N_2}^{1/2}$ and utilizing relations in (A-5), the equation (A-11) is rewritten as follows:

$$A_1 U^2 + B_1 U + C_1 = 0 \tag{A-15}$$

where

$$A_1 = 1 + (1 + \eta_4)\, K_3\, W$$

$$B_1 = K_1 + (1+\eta_4) K_6 W + (1+2\eta_4) K_9 W^2$$

$$C_1 = (1+2\eta_4) W^2 + (1+\eta_4) K_5 W - 1$$

and

$$\eta_4 = \frac{y'_{Ar}}{(y'_N + y'_{NO} + y'_{NO_2} + 2y'_{N_2O} + 2y'_{N_2})} \tag{A-16}$$

By combining equations (A-12) and (A-13) and utilizing relations in (A-5) one obtains:

$$A_2 W^2 + B_2 W + C_2 = 0 \tag{A-17}$$

where $A_2 = 2 + (2-\eta_2) K_9 U$, $B_2 = K_5 + (1-\eta_2) K_6 U + (1-2\eta_2) K_8 U^2$, $C_2 = -\eta_2 K_1 U - 2\eta_2 U^2$

and

$$\eta_2 = \frac{y'_N + y'_{NO} + y'_{NO_2} + 2y'_{N_2O} + 2y'_{N_2}}{y'_O + y'_{NO} + 2y'_{NO_2} + y'_{N_2O} + 2y'_{O_2}} \tag{A-18}$$

Equations (A-15) and (A-17) are solved simultaneously for U and W by casting them into a minimization problem and then solving the minimization problem via 2nd order Newton's Method. Finally, y_{Ar} and n_T are determined as follows:

$$y_{Ar} = \eta_4 (y_N + y_{NO} + y_{NO_2} + 2y_{N_2O} + 2y_{N_2}) \tag{A-19}$$

and

$$n_T = \frac{n'_T (y'_N + y'_{NO} + y'_{NO_2} + 2y'_{N_2O} + 2y'_{N_2})}{(y_N + y_{NO} + y_{NO_2} + 2y_{N_2O} + 2y_{N_2})} \tag{A-20}$$

Non-Reactive Fuel-Air Mixing: Fuel-air mixture as a result of non-reactive mixing of fuel (species: H and H_2) and air (species: $O, NO, NO_2, N_2O, O_2, N_2$, and Ar) is not permitted to have new chemical species. Consequently, unidirectional chemical reaction for this case is:

$$\phi_e (y'_H H + y'_{H_2} H_2) + n'_T (y'_O O + y'_N N + y'_{NO} NO + y'_{NO_2} NO_2 + y'_{N_2O} N_2O + y'_{O_2} O_2 + y'_{N_2} N_2 + y'_{Ar} Ar) \longrightarrow$$

$$n_T (y_O O + y_H H + y_N N + y_{NO} NO + y_{NO_2} NO_2 + y_{N_2O} N_2O + y_{O_2} O_2 + y_{H_2} H_2 + y_{N_2} N_2 + y_{Ar} Ar) \tag{A-21}$$

By definition,

$$y_O + y_H + y_N + y_{NO} + y_{NO_2} + y_{N_2O} + y_{O_2} + y_{H_2} + y_{N_2} + y_{Ar} = 1 \tag{A-22}$$

Conservation of masses of O , N , and Ar are identical to as stated in equations (A-12), (A-13), and (A-14). Conservation of mass of H is

$$\phi_e (y'_H + 2y'_{H_2}) = n_T (y_H + 2y_{H_2}) \tag{A-23}$$

The manipulated form of equation (A-22) is identical to that of equation (A-15) namely

$$A_1 U^2 + B_1 U + C_1 = 0 \qquad \text{(A-24)}$$

where

$$A_1 = 1 + (1+\eta_4') K_8 W$$

$$B_1 = K_1 + (1+\eta_4') K_6 W + (1+2\eta_4') K_9 W^2$$

$$C_1 = (1+2\eta_4') W^2 + (1+\eta_4') K_5 W + \frac{1}{2} y_H - 1$$

and

$$\eta_4' = \eta_4 + \frac{1}{2} \frac{\phi_e}{n_T'} \cdot \frac{(y_H' + 2y_{H_2}')}{(y_N' + y_{NO}' + y_{NO_2}' + y_{N_2O}' + 2y_{N_2}')} \qquad \text{(A-25)}$$

The equation (A-17) holds as is and by letting $V = y_{H_2}^{1/2}$ the manipulated form of equation (A-23) is

$$2V^2 + K_2 V - C_3 = 0 \qquad \text{(A-26)}$$

where $$C_3 = \frac{\phi_e}{n_T} (y_H' + 2y_{H_2}')$$

Equations (A-19) and (A-20) hold as they are.

In principle equations (A-24) and (A-17) are to be solved simultaneously for U and W however equation (A-24) contains the mole fraction y_H. An iterative solution procedure may be as follows: Make an initial guess to the value of y_H. Carry out simultaneous solutions of equations (A-24) and (A-17) and then improve this guess by using equation (A-26).

Reacting Fuel-Air Mixture Dissociation and Recombination: The reaction products of fuel-air mixtures is assumed to have following chemical species: O, H, OH, H_2O, N, NO, HO_2, NO_2, N_2O, O_2, H_2, N_2, and Ar. The unidirectional chemical reaction for this case is:

$$n_T'(y_O' O + y_H' H + y_{OH}' OH + y_{H_2O}' H_2O + y_N' N + y_{NO}' NO + y_{HO_2}' HO_2 + y_{NO_2}' NO_2 + y_{N_2O}' N_2O + y_{O_2}' O_2 + y_{H_2}' H_2 + y_{N_2}' N_2 + y_{Ar}' Ar)$$

$$\longrightarrow n_T(y_O O + y_H H + y_{OH} OH + y_{H_2O} H_2O + y_N N + y_{NO} NO + y_{HO_2} HO_2$$
$$+ y_{NO_2} NO_2 + y_{N_2O} N_2O + y_{O_2} O_2 + y_{H_2} H_2 + y_{N_2} N_2 + y_{Ar} Ar) \qquad \text{(A-27)}$$

By definition,

$$y_O + y_H + y_{OH} + y_{H_2O} + y_N + y_{NO} + y_{HO_2} + y_{NO_2} + y_{N_2O} + y_{O_2} + y_{H_2} + y_{N_2} + y_{Ar} = 1 \qquad \text{(A-28)}$$

Conservation of mass equations for N and Ar are given by equations (A-13) and (A-14) respectively. Conservation of mass equations for O and H respectively are:

$$n_T'(y_O' + y_{OH}' + y_{H_2O}' + y_{NO}' + 2y_{HO_2}' + 2y_{NO_2}' + y_{N_2O}' + 2y_{O_2}') = n_T(y_O + y_{OH} + y_{H_2O} + y_{NO} + 2y_{HO_2} + 2y_{NO_2} + y_{N_2O} + 2y_{O_2} \qquad \text{(A-29)}$$

$$n_T'(y_H' + y_{OH}' + 2y_{H_2O}' + y_{HO_2}' + 2y_{H_2}') = n_T(y_H + y_{OH} + 2y_{H_2O} + y_{HO_2} + 2y_{H_2}) \qquad \text{(A-30)}$$

Equation (A-28) is rewritten as

$$A_1 U^2 + B_1 U + C_1 = 0 \qquad \text{(A-31)}$$

where

$$A_1 = 1 + K_7 V + (1+\eta_4) K_8 W$$

$$B_1 = K_1 + (K_3 + K_4 V)V + \left[(1+\eta_4)K_6 + (1+2\eta_4)K_9 W\right] W$$

$$C_1 = (K_2 + V)V + \left[(1+\eta_4)K_5 + (1+2\eta_4)W\right] W - 1.0$$

η_4 definition is the same as in equation (A-16).

Equations (A-13) and (A-30) are combined to yield:

$$A_2 W^2 + B_2 W + C_2 = 0 \qquad \text{(A-32)}$$

where

$$A_2 = 2(1 + K_9 U)$$

$$B_2 = K_5 + K_6 U + K_8 U^2$$

$$C_2 = -\eta_0 V\left[2(1+K_4 U)V + (K_2 + K_3 U + K_7 U^2)\right]$$

and

$$\eta_0 = \frac{y_N' + y_{NO}' + y_{NO_2}' + 2y_{N_2O}' + 2y_{N_2}'}{y_H' + y_{OH}' + 2y_{H_2O}' + y_{HO_2}' + 2y_{H_2}'}$$

Similarly equations (A-29) and (A-30) are combined to yield:

$$A_3 V^2 + B_3 V + C_3 = 0 \qquad \text{(A-33)}$$

where

$$A_3 = 2 + (2 - \eta_1) K_4 U$$

$$B_3 = K_2 + \left[(1-\eta_1) K_3 + (1-2\eta_1) K_7 U\right] U$$

$$C_3 = -\eta_1 U \left[K_1 + 2U + (K_6 + 2K_8 U) W + K_9 W^2\right]$$

and

$$\eta_1 = \frac{y'_H + y'_{OH} + 2y'_{H_2O} + y'_{HO_2} + 2y'_{H_2}}{y'_O + y'_{OH} + y'_{H_2O} + y'_{NO} + 2y'_{HO_2} + 2y'_{NO_2} + y'_{N_2O} + 2y'_{O_2}}$$

Equations (A-31), (A-32), and (A-33) are solved simultaneously for U, V, and W again by casting them into a minimization problem. Finally, y_{Ar} and n_T are determined from equations (A-19) and (A-20) respectively.

APPENDIX B
AN IDEAL FUEL-AIR MIXING MODEL AT REACTOR INLET

Physically, the propulsion system does not have a component that can be identified as a mixer. However, in order to refer to various quantity ratios across the reactor with a reasonable accuracy, an ideal mixing of fuel and air is visualized to exist at reactor inlet. This mixer does not have any physical length along the direction of flow. The sketch below identifies various stations of the mixer:

The governing equations are:

State Equations:

$$p_2 = \rho_2 \frac{\mathcal{R}}{\mathcal{M}_2} T_2 \quad , \quad p_3 = \rho_3 \frac{\mathcal{R}}{\mathcal{M}_3} T_3 \quad , \quad p_4 = \rho_4 \frac{\mathcal{R}}{\mathcal{M}_4} T_4 \qquad \text{(B-1)}$$

Note that the fuel is assumed to be in gaseous state.

Conservation of Mass:

$$(1+f_e)\,\dot{m}_2' = \dot{m}_4 \qquad \text{(B-2)}$$

where $f_e = \dot{m}_3'/\dot{m}_2'$ is the effective fuel-air mass ratio.

<u>Conservation of Momentum</u>:

$$[\dot{m}_2' U_2 + A_2' p_2] + [\dot{m}_3' U_3 + A_3' p_3] = [\dot{m}_4 U_4 + A_4 p_4] \qquad \text{(B-3)}$$

Note that this mixer does not have any physical length, thus no frictional flow losses are included in the equation (B-3).

<u>Conservation of Energy</u>:

$$H(T_2, p_2, U_2) + f_e \cdot H(T_3, p_3, U_3) = (1+f_e) H(T_4, p_4, U_4) \qquad \text{(B-4)}$$

Where in above equation the conservation of mass equation (B-2) is utilized. The equations (B-1), (B-2), and (B-3) are combined to yield.

$$\mathcal{R}_0 \left\{ \frac{T_2}{\mathcal{M}_{o2} U_2} + f_e \frac{T_3}{\mathcal{M}_{o3} U_3} \right\} + (U_2 + f_e U_3) = \mathcal{R}_0 (1+f_e) \frac{T_4}{\mathcal{M}_{o4} U_4} + (1+f_e) U_4 \qquad \text{(B-5)}$$

Where in above equation we have utilized the fact that $\dot{m} = \rho U A$.

Since the reactor inlet pressure p_4 is a specified quantity the equations (B-4) and (B-5) are to be solved iteratively for T_4 and U_4 .

When fuel entering the mixer is in liquid phase, then in equation (B-3) the term $A_3' p_3$ is neglected and the following equation replaces equation (B-5):

$$\mathcal{R}_0 \frac{T_2}{\mathcal{M}_{o2} U_2} + (U_2 + f_e U_3) = \mathcal{R}_0 (1+f_e) \frac{T_4}{\mathcal{M}_{o4} U_4} + (1+f_e) U_4 \qquad \text{(B-6)}$$

When fuel speed, U_3 entering mixer is zero then $U_4 = U_2$ is used which is a degenerate form of momentum equation.

APPENDIX C
ONE DIMENSIONAL FLOW MODEL OF REACTOR

The governing equations of one dimensional flow reactor are well known (See for instance [6]) and are summarized below:

Equation of State:

$$\frac{p_5}{p_4} = \frac{\mathcal{M}_{o4}}{\mathcal{M}_{o5}}\,\frac{\rho_5}{\rho_4}\,\frac{T_5}{T_4} \tag{C-1}$$

Conservation of Mass:

$$\rho_5\, u_5\, A_5 = \rho_4\, u_4\, A_4 \tag{C-2}$$

Conservation of Momentum:

$$\rho_5 u_5^2 A_5 + p_5 A_5 = \rho_4 u_4^2 A_4 + p_4 A_4 + \frac{1}{2}(p_4+p_5)(A_5-A_4) - \frac{1}{2} C_D A_4 \left(\frac{1}{2}\rho_4 u_4^2 + \frac{1}{2}\rho_5 u_5^2\right) \tag{C-3}$$

Conservation of Complete Energy:

$$H(T_5, p_5, u_5) = H(T_4, p_4, u_4) \tag{C-4}$$

The last term on the right hand side of conservation of momentum equation is an average shearing force acting on lateral faces of reactor control surface (see for instance [7]) and C_D is a drag coefficient ($C_D = 0$ for an ideal reactor).

Equations (C-1) through (C-4) contain five unknowns: T_5, p_5, u_5, ρ_5 and A_5. The molecular weight of combustion products, $\mathcal{M}_{o5}$ is a function of p_5 and T_5. As a consequence, one of these unknowns must be specified, and in addition the equation (C-4) is reserved as if it is to be used to determine T_5 since in iterative calculation procedures T_5 is efficiently utilized as an iteration parameter.

Conveniently, one may wish to specify either pressure ratio across the reactor or area ratio across the reactor or gas speed ratio across the reactor. For this reason the following dimensionless ratios are introduced:

$$\bar{A} = A_5/A_4 \;,\quad \bar{P} = p_5/p_4 \,,\quad \bar{U} = u_5/u_4 \,,\; \text{and} \quad \bar{T}^* = \frac{T_5}{T_4}\cdot\frac{\mathcal{M}_{o4}}{\mathcal{M}_{o5}} \tag{C-5}$$

and the equations (C-1) and (C-2) combined and the equation (C-3) respectively are

$$\bar{A}\,\bar{P}\,\bar{U} = \bar{T}^* \tag{C-6}$$

and

$$\bar{U} - 1 + \frac{1}{2\gamma_4 M_4^2}(\bar{P}-1)(\bar{A}+1) + \frac{1}{4}C_D\left(1+\frac{\bar{U}}{\bar{A}}\right) = 0 \qquad \text{(C-7)}$$

An abbreviated iterative calculation procedures of equations (C-4), (C-6) and (C-7) are presented below for respectively when reactor pressure ratio, reactor cross sectional area ratio, and reactor gas speed ratio across the reactor is specified. These procedures start from the point at which a trial value of T_5 is already assigned and proper care during iterations is taken so that reactor at all times remains in supersonic gas flow state.

1. When Pressure Ratio, $\bar{P}$ is Specified

The equations (C-6) and (C-7) are rearranged and put in the following computational order:

$$\{(\bar{P}-1)\bar{P}\}\bar{A}^3 - \left\{\left[(2\gamma_4 M_4^2)\left(1-\frac{C_D}{4}\right)-(\bar{P}-1)\right]\bar{P}\right\}\bar{A}^2 + \{(2\gamma_4 M_4^2)\bar{T}^*\}\bar{A} + \left\{\left(\frac{C_D}{4}\right)(2\gamma_4 M_4^2)\bar{T}^*\right\} = 0 \qquad \text{(C-8)}$$

and

$$\bar{U} = \frac{\bar{T}^*}{\bar{A}\,\bar{P}} \qquad \text{(C-9)}$$

Note that when T_5 trial value is specified then $\bar{T}^*$ value is fixed since combustion products molecular weight $\mathcal{M}_5$ is a function of T_5 and f_5 only.

The cubic equation for $\bar{A}$, the equation (C-8), is expected to have two positive real roots or none. The value of $\bar{A}$ that corresponds to supersonic flow can be determined in a routine manner and this solution of cubic equation can be obtained via Newton-Raphson method. Final step in iterative procedure is to test whether or not trial values T_5 and U_5 satisfy the equation (C-4).

2. When Area Ratio, $\bar{A}$ is Specified

The equations (C-6) and (C-7) are rearranged and put in the following computational order:

$$\bar{P}^2 - \left\{1 + \frac{(2\gamma_4 M_4^2)}{(\bar{A}+1)}\left(1-\frac{C_D}{4}\right)\right\}\bar{P} + \left\{\frac{(2\gamma_4 M_4^2)}{(\bar{A}+1)}\left(1+\frac{C_D}{4}\frac{1}{\bar{A}}\right)\frac{\bar{T}^*}{\bar{A}}\right\} = 0 \qquad \text{(C-10)}$$

and

$$\bar{U} = \frac{\bar{T}^*}{\bar{P}\,\bar{A}} \qquad \text{(C-11)}$$

For this case when T_5 trial value is specified then $\bar{T}^*$ is not fixed since combustion products molecular weight $\mathcal{M}_5$ is also function of pressure p_5 which is to be determined from (C-10). Thus a secondary iterative procedure for p_5 is necessary. However, this particular iterative procedure need not be sophisticated since $\mathcal{M}_5$ is a weak function of p_5 .

The quadratic equation for $\bar{P}$, the equation (C-10) is expected to yield two positive real roots and the value of $\bar{P}$ corresponding to supersonic reactor is to be determined.

3. <u>When Gas Speed Ratio, $\bar{U}$ is Specified</u>

The equations (C-6) and (C-7) are rearranged and put in the following computational order:

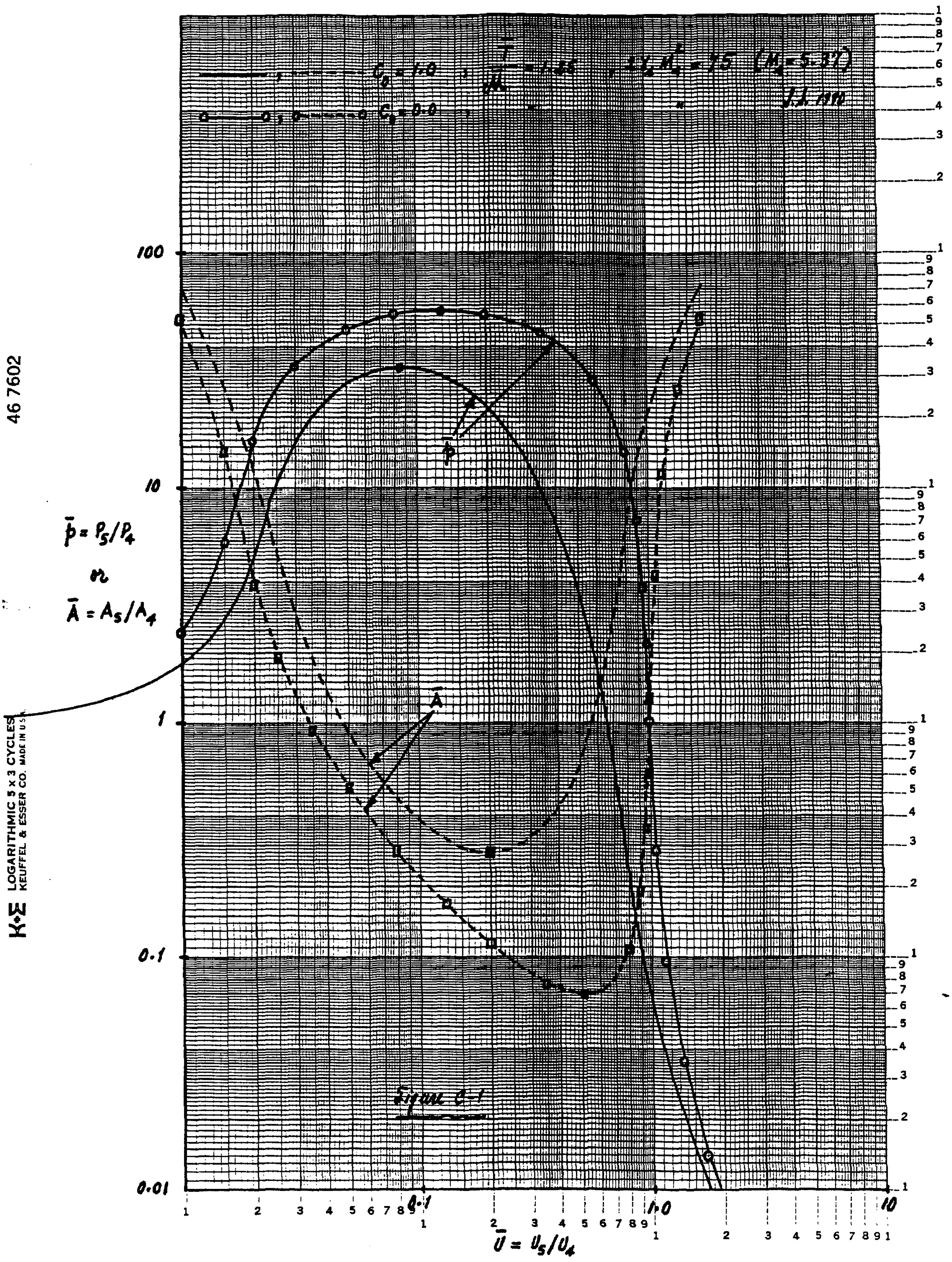
$$\left\{\left[1+\left(\frac{C_D}{4}\right)(2\gamma_4 M_4^2)\frac{\bar{U}^2}{\bar{T}^*}\right]\bar{U}\right\}\bar{P}^2+\left\{\bar{T}^*+\left[(2\gamma_4 M_4^2)\bar{U}-\left\{1+(2\gamma_4 M_4^2)\left(1-\frac{C_D}{4}\right)\right\}\right]\bar{U}\right\}\bar{P}-\bar{T}^*=0 \qquad \text{(C-12)}$$

and

$$\bar{A}=\frac{\bar{T}^*}{\bar{P}\,\bar{U}} \qquad \text{(C-13)}$$

The iterative solution procedure of this case is analogous to the case when reactor area ratio, $\bar{A}$ is specified.

Figure (C-1) presents variations of $\bar{P}$ and $\bar{A}$ with $\bar{U}$ as determined from equations (C-12) and (C-13) for a typical case that has $\bar{T}^*=1.25, 2\gamma_4 M_4^2=75$ (may correspond to $M_4=5.37$) and for values of drag coefficient, C_D = 0 and 1.0.



C_D = 1.0
C_D = 0.0
p̄ = P_5/P_4
or
Ā = A_5/A_4
Ū = U_5/U_4
p̄
Ā
100
10
1
0.1
0.01
1.0
10
46 7602
K&E LOGARITHMIC 5 x 3 CYCLES
KEUFFEL & ESSER CO. MADE IN U.S.A.

Figure C-1

APPENDIX D
A GUIDE TO USERS OF SCRAMJET PERFORMANCE ANALYSIS PROGRAM

Disclaimer:

Although this version of SCRAMJET performance analysis program, SCRAMJET.FOR, has been tested by the author of this manuscript thoroughly, no warranty, expressed or implied, is made by the author as to the accuracy and functioning of the program and related program material, nor shall the fact of distribution of them constitute and such warranty, and no responsibility is assumed by the author in connection therewith.

Remarks on Program Particulars:

1. Response to all program messages that end with a question mark, ? is either any word begins with the letter Y for YES or else for NO.

2. Program performs a limited number of checks on the validity of user entered responses. Consequently, program execution may terminate ungracefully. Program trapped error conditions shall display error messages that may not make any sense to the user. These messages shall be followed by the following message:

   Wish to carry out another performance analysis?

   A YES response to this interrogation message is a restart of program execution.

3. Stagnation temperature values may often exceed 6000K (temperature upper limit of JANAF Thermochemical Tables [2]), in such instances the following message shall appear twice during calculations:

   TEMPERATURE IS OUTSIDE OF RANGE 100-6000K

   Computations shall continue, thus user should ignore these messages.

4. Units of physical quantities to be entered by the user are clearly indicated in user prompts.

5. Results produced during program execution are displayed both in engineering and in SI units. These results are stored in disk data file, DATA.DAT automatically. The user may save this data file by responding YES to the message:

   Wish to save detailed results in DATA.DAT file?

This is the last program message that user must respond to before program execution termination.

Remarks on Program Inputs:

1. Primary inputs to SCRAMJET performance analysis program are flight Mach number, M_0^* and flight altitude, z_0^* .

2. Flight Mach number is always specified by the user.

3. Flight altitude, z_0^* is specified by the user as one of the analysis options ($5\ kft \le z_0^* \le 280\ kft$). There are two other analysis options:

   3.1 User specifies reactor inlet Mach number, M_4 .
   3.2 User specifies (free-stream) dynamic pressure, q_0^* .

   Both of these options are reduced to the determination of flight altitude, z_0^* at which specifications are met.

4. Either pressure ratio, p_5/p_4 or cross sectional area ratio, A_5/A_4 is specified across reactor.

5. Static pressure, p_2 of air at diffuser exit (or mixer inlet) is identical to that REACTOR INLET STATIC PRESSURE, p_4 .

6. Liquid hydrogen fuel tank pressure, p_H range is from 0.1 atm to 10.0 atm.

7. Phase and state of fuel at fuel energy augmenter exit may be specified in three different ways.

Option 1 Sets the state of gaseous phase fuel entering the mixer to be "proportional" to state of air entering the mixer; namely, $T_3 = r_T T_2$, $p_3 = r_p p_2$, $U_3 = r_u U_2$. r_T, r_p, r_u are proportionally constants to be specified by the user. This option does not permit heat exchanger specification.

Option 2 Sets the state of gaseous phase fuel entering the exit by specifying stagnation temperature, T_{t3} , stagnation pressure p_{t3} , and static pressure, p_3 . This option does not permit heat exchanger specification.

Option 3 User specifies phase of fuel (liquid or gas), fuel static temperature, T_{1X} , static pressure, p_{1X} , and speed, U_{1X} at fuel energy augmenter exit. In this option a heat exchanger either at diffuser exit or at reactor exit may also be specified by the user. When unmixed fuel in the combustor is in liquid phase its contribution to fuel specific impulse is neglected. Fuel bypass in liquid phase is not permissible.

8. FUEL AIR RATIO [EQUIVALENCE RATIO] either default or user specified is mixer fuel-air ratio, f_m [mixer equivalence ratio, ϕ_m].

8.1 Overall fuel-air ratio, $f_o = \dfrac{(1+\beta_R)}{(1+\beta_Q)} \cdot f_m$

where

β_R = fuel bypass ratio and β_Q = air bypass ratio.

8.2 Effective fuel-air ratio, $f_e = \dfrac{\eta_H}{\eta_A} \cdot f_m$

where η_H = fuel mixing efficiency and η_A = air mixing efficiency.

8.3 Equivalence ratio, ϕ is an alternative measure of fuel-air ratio, f

$$\phi \doteq 34.29 f$$

9. User should avoid using effective equivalence ratio, ϕ_e identically equal to unity. For good numerical accuracy ϕ_e = 0.99 or ϕ_e = 1.01 is recommended instead of ϕ_e = 1.0.

10. Range of reactor drag coefficient, C_D values is from 0.0 to 4.0.

11. PERFECTLY MIXED, UNMIXED AIR, AND UNMIXED FUEL STREAMS are introduced to simulate mixing imperfections in the combustor. In order to emphasize the fact that these three streams are non-interacting streams, each stream is assigned to have its own nozzle as depicted in Figure 1, 2, and 3. Composition of flow through each nozzle may be in chemical equilibrium or frozen chemically or composition may be forced to freeze by the user at a particular location in the nozzle as explained in the body of this report. For the latter situation the program shall display the following message:

AT THIS POINT COMPUTATIONS SWITCHED TO FROZEN COMPOSITION (U)

(U) signifies that this composition freezing was requested by user. Similar message may appear with (P) qualifier. This signifies that equilibrium composition calculations cannot be continued, in most cases it corresponds to natural composition freezing.

12. IDEAL EXPANSION refers to ideally expanding nozzle with nozzle expansion coefficient, $C_e = 1$.

    p_e = nozzle exit static pressure and $p_0 \equiv p_0^*$ = ambient air pressure at altitude, z_0^* .

    Same value of C_e is assigned to each nozzle associated with perfectly mixed, unmixed air, and unmixed fuel streams.

13. Same value of nozzle velocity coefficient, C_u is assigned to each nozzle associated with perfectly mixed, unmixed air, and unmixed fuel streams.

14. Due to Microsoft FORTRAN compiler limitations, PC version of SCRAMJET.FOR does not contain air bypass option.

Remarks on Program Outputs:

1. Area Ratios:

   AOL = A_0^* = capture cross-sectional area (free stream).

   AO = A_0 = diffuser inlet cross-sectional area.

   general form: $\dfrac{A_i}{A_j} = \dfrac{p_j}{p_i} \cdot \dfrac{\mathcal{M}_{0j}}{\mathcal{M}_{0i}} \cdot \dfrac{u_j}{u_i} \cdot \dfrac{T_i}{T_j}$

Normally, station i = element exit and station j = element inlet.
Nozzle station x = a particular location within nozzle.

Reactor area ratio and nozzle area ratios are correct for only perfectly mixed fuel and air.

2. Stagnation Temperature Ratio = T_{ti} / T_{tj}

   Station i = element exit and station j = element inlet
   Stagnation pressure ratios are defined in an analogous fashion.

3. <u>Heat Input to a Stream</u>:

Positive Value = Heat added to a stream
Negative Value = Heat removed from a stream

3.1 FUEL AT ENERGY AUGMENTER EXIT:

HEAT INPUT AT AUGMENTER is heat input to fuel stream per unit of mass of fuel, that is

$$\frac{\dot{Q}_H}{\dot{m}_H} = H(T_{1X}, p_{1X}, U_{1,X}) - H(T_H, p_H, U_H)$$

3.2 FUEL AT HEAT EXCHANGER EXIT (OR MIXER INLET):

HEAT INPUT AT HEAT EXCHANGER is heat input to the fuel stream per unit mass of fuel, that is

$$\frac{\dot{Q}_X}{\dot{m}_H} = H(T_1, p_1, U_1) - H(T_{1X}, p_{1X}, U_{1X})$$

This heat input is displayed whether a heat exchanger is present or not.

3.3 MIXER INLET:

HEAT INPUT AT DIFFUSER EXIT is heat input to air stream per unit mass of air, that is

$$-\frac{\dot{Q}_X}{\dot{m}_2} = H(T_2, p_2, U_2) - H(T_{2X}, p_{2X}, U_{2X})$$

MIXER INLET quantities are displayed only when heat exchanger at diffuser exit is present.

3.4 PERFECTLY MIXED REACTOR STREAM NOZZLE INLET:

HEAT INPUT AT REACTOR EXIT is heat input to combustion products per unit mass of combustion products, that is

$$-\frac{\dot{Q}_X}{\dot{m}_5} = H(T_5, p_5, U_5) - H(T_{5X}, p_{5X}, U_{5X})$$

PERFECTLY MIXED REACTOR STREAM NOZZLE INLET quantities are displayed only when heat exchanger at reactor exit is present.

4. LOCAL SPECIFIC THRUST, $T_{sp} = T / (\dot{m}_0^* a_0^*)$

Also note:

$$T_{sp} = (I_{sp})_{air} / a_0^* = \frac{1}{2} M_0^* C_F$$

where $C_F = T / (q_0^* A_0^*)$ is the conventional thrust coefficient.

Program Error Messages:

It is questionable that SCRAMJET performance analysis program can handle all conceivable analysis cases that the user may wish to try. Consequently, the program execution shall terminate, occasionally, ungracefully. In such instances, the user should keep an accurate account of particular cases that cause ungraceful program execution termination and transmit that information to the author.*

A limited effort was made to trap some numerical procedure irregularities. In such instances, there shall appear program error message(s) and program execution of a particular case shall terminate gracefully which is acknowledged by the following message:

Wish to carry out another performance analysis?

Some program displayed error messages may not have any meaning to the user and their explanations are beyond the scope of this report.

Below are some program displayed error messages and their probable causes:

1. TEMPERATURE IS OUTSIDE OF RANGE 100-6000K

Appearance of this message during stagnation temperature calculations is normal and user should ignore them since program execution shall continue. This message shall also be displayed when temperature range outside of JANAF Thermochemical Tables [2], thus an iterative solution procedure fails.

* Harvey Mudd College, Claremont, CA 91711 . Phone: (909) 621-8000, Ext. 3296

2. TEMPERATURE ITERATIONS FAILED!!!

Temperature is a primary variable in number of iterative procedures. This message simply signifies that iterative procedure used fails to find a solution. It may not mean that the problem at hand is an ill posed problem.

3. AT DIFFUSER EXIT STAGNANT STATE ENCOUNTERED!!!

Flow speed at diffuser exit must be greater than zero. Increasing flight Mach number, M_0^* or decreasing flight altitude, z_0^* may remedy this problem.

4. FLIGHT ALTITUDE RANGE IS FROM 5 TO 280 kft!!!

Either user managed to specify an altitude value outside of this range or for some performance analysis options, when program tries to adjust flight altitude in order to meet specified dynamic pressure, q_0^* or combustor inlet Mach number, M_4 altitude range of U.S. Standard Atmosphere Tables [1] is exceeded. Readjust flight analysis specifications.

5. PRESSURE CANNOT BE LESS THAN 1.0E-20 (atm)!!!

Program does not allow any pressure value identically equal to zero. The appearance of this message is extremely rare.